令和 5 年度　博士論文

Disturbance Observers for Periodic Disturbance

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Chapter 1

Introduction

1.1 Motivations

This thesis started with a simple example to illustrate the major motivation of this thesis. To consider the following system:

$$\begin{cases} \dot{x} &= -Ax + Bu + d \\ y &= Cx \end{cases}, \tag{1.1}$$

where u the control input, x the state, y the interested controlled output, A, B and C the system parameters, and d the disturbance. Let y_r the desired value that the output is expected to achieve, which is usually called setpoint or object value. Without loss of generality, the setpoint value y_r is taken as a constant one in this example for simplicity, that is, $\dot{y}_r = 0$.

Defining the tracking error variable of the system as $e_y = y_r - y$, system (1.1) is equivalently depicted by

$$\begin{cases} \dot{e}_y &= -Ae_y - Bu - d + Ay_r \\ y &= y_r - e_y \end{cases}, \tag{1.2}$$

The control object here is to propose the parameterization of all disturbance observers for periodic disturbances and to design a control law u in terms of the tracking error and the setpoint, that is, $u = u(e_y, y_r)$, such that the real output y achieves its desired setpoint y_r. Therefore, the tracking error e_y tends to zero as time goes to infinity.

1.2 Basic Framework

A basic framework of disturbance observer based control method [1] is shown in Figure 1.1. As shown in this

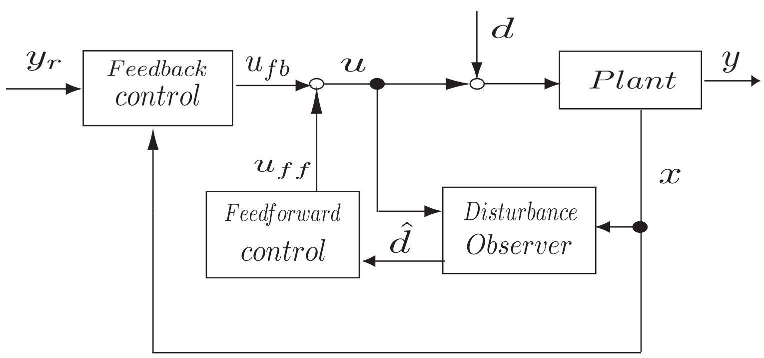

Fig. 1.1: A basic framework of disturbance observer based control

figure, the composite controller consists of two parts: a feedback control part and a feedforward control part based on a disturbance observer. The feedback control is generally employed for tracking and stabilization of the nominal dynamics of the controlled plant. In this stage, the disturbances and uncertainties are not necessarily required to be considered. The disturbances and uncertainties on controlled plant are estimated by a disturbance observer and then compensated by a feedforward control. The major merit of such design lies in that the feedback control and feedforward designs satisfy the so-called separation principal, that is, the tracking control performance and the disturbance rejection performance can be achieved by adjusting the feedback and feedforward controllers, respectively. Such a promising feature has induced many superiorities as compared with the passive antidisturbance control methods.

1.3 A Trend of A Study for The Parameterization of All Disturbance Observers

In this thesis, examine the parameterization of all disturbance observers for periodic disturbances. A disturbance observer is used to estimate the disturbances in the factory plant [2, 3, 4, 5, 6, 7, 8, 9]. Several papers on design methods for disturbance observers have been publishing [5, 6, 7, 10, 11]. Currently, the applications of disturbance observers have been using in many control systems such as a motion-control field [4, 6, 12, 13]. A disturbance observer is used in motion control to cancel the disturbance or to make the closed-loop system robustly stable [2, 3, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20]. Typically disturbance observers include disturbance signal generators and an observer. Disturbances are normally considered step disturbances are estimated by the observer. Since the disturbance observer is simple to understand the structure, and it is used in many cases [2, 3, 11, 12, 14, 15, 16, 17].

Mita et al. point out that disturbance observers are not the only alternative design of complete controllers [10]. That is, a control system with a disturbance observer does not guarantee robust stability. Extended H_∞ control in [10] presented an effective motion control method that cancels disturbances. To confirm the method in [10], can be used a control system with a disturbance observer could be designed to guarantee robust stability. From another point of view, Kobayashi et al. considered an observer design method for obtaining phase compensation based on disturbance observers [11]. Comparing to use a phase compensator, the control system in [11] is simple and easy to design. In this way, a robustness analysis of the control system with disturbance observer has been considered.

Another important control problem is the parameterization problem which is the problem of finding all stable controllers for the plant [21, 22, 23, 24, 25, 26, 27, 28]. If the parameterization of all disturbance observers for any disturbances could be obtained, we could express results from previous studies of disturbance observers in a uniform manner. In addition, disturbance observers for any disturbances could be designed systematically. From this point of view, Yamada et al. examine parameterizations of all disturbance observers and all linear functional disturbance observers for plants with any input-output disturbances [29, 30, 31, 32, 33]. Ando et al. examine parameterizations of all disturbance observers and all linear functional disturbance observers for plants with any input and output disturbances [34]. Phukapak et al. overcome this problem and examine the parameterizations of all disturbance observers for periodic output disturbances [35]. In addition, the previous study [36] explained that the parameterization of all disturbance observers for periodic input disturbances and that of disturbance observers for periodic input and output disturbances [37].

1.4 The Purpose and Contents of This Study

In this thesis is expanded the result in [35, 36, 37] and purpose of this study is to propose the parameterization of all disturbance observers for periodic disturbances.

Methods in [29, 30, 31, 32, 33, 34] can estimate disturbances with finite number of frequency component but cannot estimate disturbances with infinite number of frequency component. In addition, when we control practical systems, many disturbances appear as periodic disturbances, such as robot arms, heat-flow experiments, multi-axis manipulators, positioning system, noises and vibrations [4, 33, 38, 39, 40]. Therefore, it is important for the control system to attenuate periodic disturbances. In addition, the parameterization motivated by the above, parameterization and disturbance observers are taken into account for periodic disturbances in this research. Based on a control system to attenuate periodic disturbances strategy is proposed to improve the performance of the parameterization of all disturbance observers for periodic disturbances. For these reasons, the purpose of this study is to propose the parameterization of all disturbance observers for periodic disturbances. This study is organized following:

In chapter 2, this thesis clarifies that the periodic output disturbances could be estimated using disturbance observers and propose the parameterization of all disturbance observers for periodic output disturbances. First, the necessary structure and characteristics of disturbance observers for periodic output disturbances are defined. In addition, the problem considered in this research is explained. The conditions to estimate the periodic output disturbances are clarified. The parameterization of all disturbance observers for periodic output disturbances and that of all linear functional disturbance observers for periodic output disturbances are clarified. In addition, a design method for the linear functional disturbance observer and a procedure for linear functional disturbance observers for periodic output disturbances are clarified. Finally, the study offers a numerical example to illustrate the features of the proposed design method.

In chapter 3, this thesis clarifies that the periodic input disturbances could be estimated using disturbance observers and propose the parameterization of all disturbance observers for periodic input disturbances. First, the necessary structure and characteristics of disturbance observers for periodic input disturbances are defined. In addition, the problem considered in this chapter is explained. The conditions to estimate the periodic input disturbances are clarified. The parameterization of all disturbance observers for periodic input disturbances and that of all linear functional disturbance observers for periodic input disturbances are clarified. In addition, a design method for the linear functional disturbance observer and a design procedure for linear functional disturbance observers for periodic input disturbances are clarified. Finally, the study offers a numerical example to illustrate the features of the proposed design method.

In chapter 4, this thesis proposes the parameterization for disturbance observers for periodic input and output disturbances. First, the necessary structure and characteristics of disturbance observers and the linear functional disturbance observer for periodic input and output disturbances are introduced. Next, to attenuate periodic input and output disturbances effectively, a design method for a control system using these parameterizations of all disturbance observers and of all linear functional disturbance observers for periodic input and output disturbance is proposed. In addition, control characteristics of control system using these parameterizations are clarified. A design procedure is also given. Finally, the study offers a numerical example to illustrate the features of the proposed design method.

In chapter 5, gives concluding remarks.

Notations

R	the set of real numbers.
$R(s)$	the set of real rational functions with s.
RH_∞	the set of stable proper real rational functions.
$\mathcal{U}$	the unimodular procession in RH_∞. That is, $P(s) \in \mathcal{U}$ means that $P(s) \in RH_\infty$ and $P^{-1}(s) \in RH_\infty$.
$\bar{\sigma}(\{\cdot\})$	largest singular value of $\{\cdot\}$.
diag $(a_1, \cdots, a_n)$	an $n \times n$ diagonal matrix with a_i as its $i-th$ diagonal element.
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	represents the state space description $C(sI-A)^{-1}B+D$.
$\mathcal{L}\{\cdot\}$	the Laplace transformation of $\{\cdot\}$.

Chapter 2

Disturbance Observers for Periodic Output Disturbances

2.1 Introduction

In this chapter, the study clarifies that the periodic output disturbances could be estimated using disturbance observers and propose the parameterization of all disturbance observers for periodic output disturbances. First, the necessary structure and characteristics of disturbance observers for periodic output disturbances are defined. In addition, the problem considered in this research is explained. Conditions to estimate the periodic output disturbances are clarified. The parameterization of all disturbance observers for periodic output disturbances and that of all linear functional disturbance observers for periodic output disturbances are clarified. In addition, a design method for the linear functional disturbance observer and a procedure for linear functional disturbance observers for periodic output disturbances are clarified. Finally, the study offers a numerical example to illustrate the features of the proposed design method. This chapter is organized following: In Section 2.2, the study formulates the problem considered in this thesis. In Section 2.3, the study clarifies the conditions to estimate the periodic output disturbances. In Section 2.4, the study proposes the parameterization of all disturbance observers for periodic output disturbances. In Section 2.5, the study defines the parameterization of all linear functional disturbance observers for periodic output disturbances. In Section 2.6, the study shows a design method for the linear functional disturbance observer. In Section 2.7, the study presents a procedure for linear functional disturbance observers for periodic output disturbances. In Section 2.8, the study provides a numerical example to illustrate the features of the proposed method. In Section 2.9 gives concluding remarks.

2.2 Problem Formulation

Consider the plant described by

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + Bu(t) \\ y(t) & = & Cx(t) + d(t) \end{array} \right. , \tag{2.1}$$

where $x \in R^n$ is the state variable, $u \in R^p$ is the control input, $y \in R^m$ is the output, $d \in R^m$ is the periodic output disturbance with period $T > 0$ satisfying

$$d(t+T) = d(t)\,(\forall t \geq 0), \tag{2.2}$$

$A \in R^{n \times n}$, $B \in R^{n \times p}$ and $C \in R^{m \times n}$. It is assumed that (A, B) is stabilizable, (C, A) is detectable, A has no eigenvalue on the imaginary axis and $u(t)$ and $y(t)$ are available, but $d(t)$ is unavailable. The transfer function from $u(s)$ to $y(s)$ in (2.1) is denoted by

$$y(s) = G(s)u(s) + d(s), \tag{2.3}$$

where

$$G(s) = C(sI - A)^{-1}B \quad \in R^{m \times p}(s). \tag{2.4}$$

When the disturbance $d(t)$ is unavailable, a disturbance estimator called the disturbance observer is frequently used. The disturbance observer estimates the disturbance $d(t)$ using the available measurements. Since the available measurements of the plant in (2.1) are $u(t)$ and $y(t)$ and the disturbance $d(t)$ satisfies (2.2), the periodic output disturbance $d(t)$ is estimated by the form in

$$\tilde{d}(s) = F_1(s)e^{-sT}y(s) + F_2(s)e^{-sT}u(s), \tag{2.5}$$

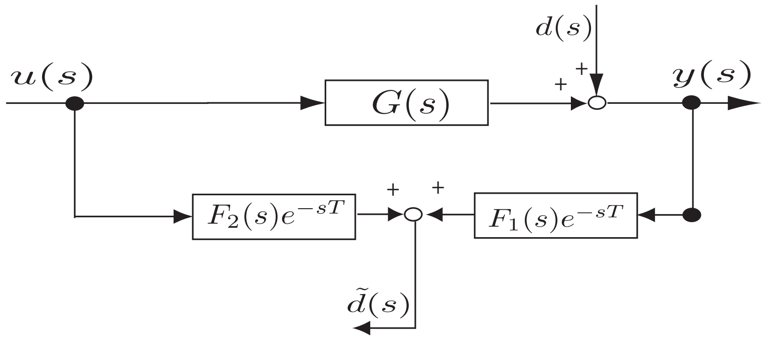

Fig. 2.1: Structure of a disturbance observer

where $F_1(s) \in RH_\infty^{m\times m}$, $F_2(s) \in RH_\infty^{m\times p}$, $\tilde{d}(s) = \mathcal{L}\{\tilde{d}(t)\}$ and $\tilde{d}(t) \in R^m$. The structure of disturbance observer $\tilde{d}(s)$ in (2.5) is shown in Fig. 2.1.
The concept of a disturbance observer for periodic output disturbances is proposed following:

Definition *(disturbance observer for periodic output disturbances)*
The study calls the system $\tilde{d}(s)$ in (2.5) a "disturbance observer for periodic output disturbances", if the error $e(t)$ between $d(t)$ and $\tilde{d}(t)$ written by

$$e(t) = d(t) - \tilde{d}(t) \tag{2.6}$$

satisfies

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} \left(d(t) - \tilde{d}(t)\right) = 0 \tag{2.7}$$

for any initial state $x(0)$, control input $u(t)$ and periodic output disturbance $d(t)$.

If the parameterization of all disturbance observers for periodic output disturbances is obtained, there is a possibility to attenuate periodic output disturbances without using the repetitive control system. In addition, the study can design the disturbance observers for periodic output disturbances systematically. However, no research examines the parameterization of all disturbance observers for periodic output disturbances.

The problem considered in this thesis is to propose the parameterization of all disturbance observers for periodic output disturbances. That is, the study obtains the parameterization of all disturbance observers for $\tilde{d}(s)$ in (2.5) for periodic output disturbances.

2.3 Conditions to Estimate The Periodic Output Disturbances

In this section, the study clarifies the conditions of $\tilde{d}(s)$ in (2.5) to satisfy (2.7).
The condition of $\tilde{d}(s)$ in (2.5) satisfying (2.7) is summarized in the following theorem.

Theorem 2.3.1 *$\tilde{d}(s)$ in (2.5) works as a disturbance observer for periodic output disturbances if and only if*

$$F_1(s)N(s) + F_2(s)D(s) = 0. \tag{2.8}$$

and

$$\left(1 - e^{-s_iT}\right) e(s_i) = 0 \ \ \forall s_i (i = 0, 1, \ldots), \tag{2.9}$$

respectively, where

$$s_i = j\omega_i \tag{2.10}$$

and

$$\omega_i = \frac{2\pi i}{T} \ \ (i = 0, 1, \ldots) \tag{2.11}$$

and j is the imaginary unit.

Proof: First, the necessity is shown, that is, if $\tilde{d}(s)$ in (2.5) satisfies (2.7), then (2.8) and (2.9) are satisfied. $\tilde{d}(s)$ in (2.5) is rewritten as

$$\tilde{d}(s) = \left(F_1(s)e^{-sT}N(s) + F_2(s)e^{-sT}D(s)\right)\xi(s) + F_1(s)e^{-sT}d(s), \tag{2.12}$$

where $\xi(s)$ is the pseudo-state variable satisfying

$$u(s) = D(s)\xi(s), \tag{2.13}$$

$N(s) \in RH_\infty^{m\times p}$ and $D(s) \in RH_\infty^{m\times m}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = N(s)D^{-1}(s). \tag{2.14}$$

$\xi(s)$ in (2.12) is factorized as

$$\begin{aligned}\xi(s) &= \tilde{\xi}(s) + \bar{\xi}(s)\\ &= \frac{1}{1-e^{-sT}}\tilde{\xi}(s) + \bar{\xi}(s),\end{aligned} \tag{2.15}$$

where $\tilde{\xi}(s)$ is denoted by

$$\tilde{\xi}(s) = \int_0^T e^{-s\tau}\xi(\tau)d\tau, \tag{2.16}$$

$\tilde{\xi}(s)/(1-e^{-sT})$ means the periodic signal with period T and $\bar{\xi}(s)$ includes all other signals. In addition, $d(s)$ in (2.12) satisfying (2.2) is rewritten by

$$d(s) = \frac{1}{1-e^{-sT}}\hat{d}(s), \tag{2.17}$$

where

$$\hat{d}(s) = \int_0^T e^{-s\tau}d_i(\tau)d\tau. \tag{2.18}$$

Then the Laplace transformation of the error $e(t)$ in (2.6) is written by

$$\begin{aligned}e(s) &= \left(I - F_1(s)e^{-sT}\right)\frac{1}{1-e^{-sT}}\hat{d}(s) - \left(F_1(s)N(s) + F_2(s)D(s)\right)\frac{e^{-sT}}{1-e^{-sT}}\tilde{\xi}(s)\\ &\quad + \left(F_1(s)N(s) + F_2(s)D(s)\right)e^{-sT}\bar{\xi}(s).\end{aligned} \tag{2.19}$$

From the assumption that $e(t)$ satisfies (2.7) for any $\bar{\xi}(s)$,

$$\left(F_1(s)N(s) + F_2(s)D(s)\right)e^{-sT}\bar{\xi}(s) = 0 \tag{2.20}$$

is satisfied for any $\bar{\xi}(s)$. That is, the study has (2.8). Substitution of (2.8) for (2.19) gives

$$e(s) = \left(I - F_1(s)e^{-sT}\right)\frac{1}{1-e^{-sT}}\hat{d}(s) - \left(F_1(s)N(s) + F_2(s)D(s)\right)\frac{e^{-sT}}{1-e^{-sT}}\tilde{\xi}(s). \tag{2.21}$$

From the assumption that $e(t)$ satisfies (2.7) and internal model principle [41], (2.9) is satisfied. The study has thus proved the necessity.

Next, the sufficiency is shown. That is, if (2.8) and (2.9) are satisfied, then $e(s)$ in (2.6) satisfies (2.7). From (2.8), $e(s)$ in (2.6) is written by

$$e(s) = \left(I - F_1(s)e^{-sT}\right)\frac{1}{1-e^{-sT}}\hat{d}(s) - \left(F_1(s)N(s) + F_2(s)D(s)\right)\frac{e^{-sT}}{1-e^{-sT}}\tilde{\xi}(s). \tag{2.22}$$

From (2.9), $e(t)$ in (2.6) satisfies (2.7). Thus the sufficiency is shown.

The study has thus proved Theorem 2.3.1.

Note that from Theorem 2.3.1, when (2.8) is a condition of $\tilde{d}(s)$ disturbance observers for any state variable. In addition, (2.9) is a condition to estimate any periodic signals. Therefore, this is the most important condition to estimate the periodic output disturbances and the mentioned condition could solve the problem.

In this section, the study obtained the conditions to estimate the periodic output disturbances. In the next section, using the result of Theorem 2.3.1, the study clarifies the parameterization of all disturbance observers for periodic output disturbances.

2.4 Parameterization of All Disturbance Observers for Periodic Disturbances

In section, the study proposes the parameterization of all disturbance observers $\tilde{d}(s)$ in (2.5) for periodic output disturbances.

The parameterization is summarized in the following theorem.

Theorem 2.4.1 *The system $\tilde{d}(s)$ in (2.5) is the disturbance observer for periodic output disturbances if and only if $F_1(s)$ and $F_2(s)$ are written by*

$$F_1(s) = \tilde{D}(s) + Q(s)\tilde{D}(s) \tag{2.23}$$

and

$$F_2(s) = -\tilde{N}(s) - Q(s)\tilde{N}(s) \in RH_\infty^{m\times p}, \tag{2.24}$$

where $\tilde{D}(s) \in RH_\infty^{m\times m}$ and $\tilde{N}(s) \in RH_\infty^{m\times p}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \tilde{D}(s)^{-1}\tilde{N}(s), \tag{2.25}$$

respectively. Here $Q(s) \in RH_\infty$ is any function satisfying

$$\tilde{D}(s_i) + Q(s_i)\tilde{D}(s_i) = I \quad \forall s_i (i = 0, 1, \ldots). \tag{2.26}$$

Proof of Theorem 2.4.1 requires the following lemma.

Lemma 2.4.1 *[24] Assume that $A(s) \in RH_\infty^{m\times n}$, $B(s) \in RH_\infty^{q\times p}$, $C(s) \in RH_\infty^{m\times p}$ and*

$$\text{rank} \begin{bmatrix} A^T(s) & B^T(s) \end{bmatrix} = \gamma. \tag{2.27}$$

are satisfied. There exist $X(s) \in RH_\infty^{m\times m}$ and $Y(s) \in RH_\infty^{m\times q}$ satisfying

$$X(s)A(s) + Y(s)B(s) = C(s) \tag{2.28}$$

if and only if there exists $U(s) \in \mathcal{U}$ satisfying

$$\begin{bmatrix} A(s) \\ B(s) \\ C(s) \end{bmatrix} = U(s) \begin{bmatrix} A(s) \\ B(s) \\ 0 \end{bmatrix}. \tag{2.29}$$

When $X_0(s) \in RH_\infty^{m\times m}$ and $Y_0(s) \in RH_\infty^{m\times q}$ are solutions of (2.28), then all solutions of (2.28) are given by

$$\begin{bmatrix} X(s) & Y(s) \end{bmatrix} = \begin{bmatrix} X_0(s) & Y_0(s) \end{bmatrix} + Q(s) \begin{bmatrix} W_1(s) & W_2(s) \end{bmatrix}, \tag{2.30}$$

where $W_1(s)$ and $W_2(s)$ satisfy

$$W_1(s)A(s) + W_2(s)B(s) = 0 \tag{2.31}$$

and

$$\text{rank} \begin{bmatrix} W_1(s) & W_2(s) \end{bmatrix} = n + q - \gamma \tag{2.32}$$

and $Q(s) \in RH_\infty^{p\times(n+q-\gamma)}$ is any function.

Using Theorem 2.3.1 and Lemma 2.4.1, Theorem 2.4.1 is proved.
Proof: From Theorem 2.3.1, $\tilde{d}(s)$ works as a disturbance observer for periodic output disturbances if and only if $F_1(s) \in RH_\infty^{m\times m}$ and $F_2(s) \in RH_\infty^{m\times p}$ satisfy (2.8). From Lemma 2.4.1, all solutions $F_1(s)$ and $F_2(s)$ of (2.8) are given by (2.23) and (2.24), respectively, since

$$\tilde{D}(s)N(s) - \tilde{N}(s)D(s) = 0, \tag{2.33}$$

and Lemma 2.4.1, where $\tilde{D}(s) \in RH_\infty^{p\times p}$ and $\tilde{N}(s) \in RH_\infty^{p\times m}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \tilde{D}^{-1}(s)\tilde{N}(s). \tag{2.34}$$

The rest is to prove $\tilde{d}(s)$ in (2.5) works as a disturbance observer for periodic output disturbances if and only if $Q(s)$ in (2.23) and (2.24) satisfy (2.26). From Theorem 2.3.1, $\tilde{d}(s)$ in (2.5) works as a disturbance observer for periodic output disturbances if and only if $e(s)$ in (2.6) satisfies (2.9).

The necessity is shown. that is if $\tilde{d}(s)$ in (2.5) works as a disturbance observer for periodic output disturbances, then $Q(s)$ in (2.23) and (2.24) satisfy (2.26). From (2.23) and (2.24), $e(s)$ in (2.6) is written by

$$e(s) = \left\{I - F_1(s)e^{-sT}\right\}\frac{1}{1-e^{-sT}}\hat{d}(s). \tag{2.35}$$

This equation yields

$$\begin{aligned}\left(1-e^{-s_iT}\right)e(s_i) &= \left\{I - \left(\tilde{D}(s_i) + Q(s_i)\tilde{D}(s_i)\right)\right\}\tilde{d}(s_i) \\ &= 0. \end{aligned} \tag{2.36}$$

The study has (2.26). Thus the study proved the necessity.

Next, sufficiency is shown. That is, the study shows that if $Q(s)$ in (2.23) and (2.24) satisfy (2.26), then (2.9) is satisfied. $e(s)$ in (2.6) is written by (2.35). Substituting (2.26) to (2.35), it is obvious that (2.9) is satisfied. In this way, the sufficiency has been proved.

From the above discussion, the study has thus proved Theorem 2.4.1. Note that from Theorem 2.4.1, when $G(s)$ is stable, if $Q(s)$ is settled by

$$Q(s) = \tilde{D}^{-1}(s) - I, \tag{2.37}$$

then $Q(s)$ in (2.37) satisfies (2.26). However, when $G(s)$ is unstable, it is difficult to set $Q(s)$ satisfying (2.26). For the unstable plant $G(s)$, a disturbance observer for periodic output disturbances is often used to attenuate disturbances effectively in [42], even if the system $\tilde{d}(s)$ in (2.5) satisfying (2.9) could not be designed. This means that in order to attenuate periodic disturbances, it is enough to estimate $(I - F(s))\tilde{d}(s)$, where $F(s) \in RH_\infty$ is any function. From this point of view, in the next section, when $G(s)$ is unstable, the study defines a linear functional disturbance observer for periodic output disturbances and clarifies the parameterization of all linear functional disturbance observers for periodic output observers.

2.5 Parameterization of All Linear Functional Disturbance Observers for Periodic Output Disturbances

In this section, the study defines a linear functional disturbance observer and presents the parameterization of all linear functional disturbance observers for periodic output disturbances.

The study calls $\tilde{d}(s)$ in (2.5) the linear functional disturbance observer for periodic output disturbances if $\tilde{d}(s)$ is written by

$$\left(1-e^{-s_iT}\right)e(s_i) = F(s_i)\hat{d}(s_i) \tag{2.38}$$

is satisfied, where $F(s) \in RH_\infty^{m\times m}$ is any function satisfying

$$\bar{\sigma}\left\{F(s_i)\right\} \simeq 0 \ \ \forall s_i (i = 0, 1, \ldots, n_{max}) \tag{2.39}$$

and n_{max} is the maximum frequency satisfying (2.39). Since the available measurements of the plant $G(s)$ in (2.1) are $u(t)$ and $y(t)$ and the disturbance $d(t)$ satisfies (2.2), the periodic disturbance $d(t)$ is estimated by the form in (2.5), where $F_1(s) \in RH_\infty^{m\times m}$ and $F_2(s) \in RH_\infty^{m\times p}$.

The parameterization of all linear functional disturbance observers for periodic output disturbances is summarized in the following theorem.

Theorem 2.5.1 *The system $\tilde{d}(s)$ in (2.5) is the linear functional disturbance observer for periodic output disturbances if and only if $F_1(s)$, $F_2(s)$ and $F(s)$ are described by*

$$F_1(s) = \tilde{D}(s) + Q(s)\tilde{D}(s), \tag{2.40}$$

$$F_2(s) = -\tilde{N}(s) - Q(s)\tilde{N}(s), \tag{2.41}$$

and

$$\begin{aligned} F(s) &= I - F_1(s) \\ &= I - \left(\tilde{D}(s) + Q(s)\tilde{D}(s)\right), \end{aligned} \tag{2.42}$$

respectively, where $Q(s) \in RH_\infty^{m\times m}$ is any function satisfying

$$\bar{\sigma}\left\{I - F_1(s_i)\right\} = \bar{\sigma}\left\{I - \left(\tilde{D}(s_i) + Q(s_i)\tilde{D}(s_i)\right)\right\} \simeq 0 \ \ \forall s_i (i = 0, 1, \ldots, n_{max}). \tag{2.43}$$

Proof: First, the necessity is shown. That is, the study shows that if the system $\tilde{d}(s)$ in (2.5) is a linear functional disturbance observer for periodic output disturbances, then (2.40), (2.41), (2.42) and (2.43) are satisfied. From (2.5), (2.12), (2.13), (2.14), (2.15) and (2.16), for the system $\tilde{d}(s)$ in (2.5), $e(s)$ is written as (2.19). From the assumption that $e(s)$ satisfies (2.7) for any $\bar{\xi}(s)$, (2.21) holds for any $\bar{\xi}(s)$. That is, the study has (2.9). From (2.33) and Lemma 2.4.1, all solutions of $F_1(s)$ and $F_2(s)$ to satisfy (2.9) are given by (2.40) and (2.41), respectively. Substitution of (2.9) to (2.21) gives (2.22). From (2.22) the assumption that $e(s)$ satisfies (2.38), the study has (2.42) and (2.43). In this way, the necessity has been proved.

Next, sufficiency is shown. That is, the study shows that if (2.40), (2.41), (2.42) and (2.43) are satisfied, then the $\tilde{d}(s)$ is a linear functional disturbance observer. Since $e(s)$ in (2.6) is written by (2.19), Substituting (2.40), (2.41), (2.42) and (2.43) to (2.19), it is obvious that (2.38) is satisfied. In this way, the sufficiency has been proved.

From the above, the study has thus proved Theorem 2.5.1. Note that from Theorem 2.5.1, $\tilde{d}(s)$ satisfies (2.38) and (2.39), then (2.40), (2.41), (2.42) and (2.43) are satisfied to be solved by Theorem 2.5.1.

2.6 Design Method for Linear Functional Disturbance Observers

In this section, the study shows a design method for a linear functional disturbance observer $\tilde{d}(s)$.

In order to design the linear functional disturbance observer $\tilde{d}(s)$ for periodic output disturbances, $Q(s)$ in (2.40) and (2.41) need to satisfy (2.43).

When $G(s)$ is unstable, $Q(s)$ is set as

$$Q(s) = \hat{Q}(s)\left(I - \tilde{D}(s)\right)\tilde{D}_o^{-1}(s), \tag{2.44}$$

where $\tilde{D}_o(s) \in RH_\infty^{m\times m}$ is an outer function of $\tilde{D}(s)$ satisfying

$$\tilde{D}(s) = \tilde{D}_o(s)\tilde{D}_i(s), \tag{2.45}$$

$\tilde{D}_i(s) \in RH_\infty^{m\times m}$ is a co-inner function of $\tilde{D}(s)$ satisfying $\tilde{D}_i(0) = I$ and $\tilde{D}_i(s)\tilde{D}_i(-s)^T = I$, $\hat{Q}(s) \in RH_\infty^{m\times m}$ is any function satisfying

$$\bar{\sigma}\left\{I - \hat{Q}(s_i)\tilde{D}_i(s_i)\right\} \simeq 0 \ \ \forall s_i (i = 0, 1, \dots, n_{max}). \tag{2.46}$$

From the above, the study showed a design of the linear functional disturbance observer $\tilde{d}(s)$ for periodic output disturbances, $Q(s)$ in (2.44) satisfied (2.45) and (2.46) based on Theorem 2.5.1. When $Q(s)$ in (2.44) is designed using the method described In Section 2.7.

2.7 Procedure for Linear Functional Disturbance Observers for Periodic Output Disturbances

In this section, the study shows a design procedure for linear functional disturbance observer for periodic output disturbances satisfying Theorem 2.5.1.

A design procedure is summarized following:

<u>Procedure</u>

Step 1) Obtain coprime factors $\tilde{N}(s) \in RH_\infty^{m\times p}$ and $\tilde{D}(s) \in RH_\infty^{m\times m}$ of $G(s) \in R(s)^{m\times p}$ satisfying (2.25). The parameterization of all linear functional disturbance observers is given by (2.5), where $F_1(s)$, $F_2(s)$ and $F(s)$ are written by (2.40), (2.41) and (2.42), respectively.

Step 2) The maximum frequency range n_{max} in (2.43) to estimate the periodic disturbance $d(s)$ is settled.

Step 3) Factorize $\tilde{D}(s)$ as (2.45) satisfying $\tilde{D}_i(0) = I$.

Step 4) Settle $Q(s) \in RH_\infty^{m\times m}$ satisfying (2.43). In order to satisfy (2.43), $Q(s) \in RH_\infty^{m\times m}$ is set according to (2.44). Where $\hat{Q}(s)$ is a low-pass filter satisfying $\hat{Q}(0) = I$, as

$$\hat{Q}(s) = \text{diag}\left\{\frac{k_1}{(1 + s\tau_1)^{\alpha_1}}, \dots, \frac{k_m}{(1 + s\tau_m)^{\alpha_m}}\right\}, \tag{2.47}$$

$\alpha_i (i = 1, 2, \dots, m)$ is an arbitrary positive integer and $k_i (i = 1, 2, \dots, m)$ satisfying $\tau_i (i = 1, \dots, m)$

$$\sigma\left\{I - \hat{Q}(s_i)\tilde{D}_i(s_i)\right\} \simeq 0. \ \ \forall s_i (i = 1, \dots, m). \tag{2.48}$$

are real numbers.

Step 5) Substituting $Q(s)$ for (2.40), (2.41) and (2.42), $F_1(s)$, $F_2(s)$ and $F(s)$ are obtained. Then the study could design disturbance observer $\tilde{d}(s)$ for periodic output disturbances as (2.5).

In this section, the study showed a procedure for linear functional disturbance observers for periodic output disturbances.

2.8 Numerical Example

In this section, the study shows numerical examples to illustrate the effectiveness of the proposed parameterizations.

Firstly, the study shows that the proposed design method of the disturbance observer for the stable plant in this paper could estimate the periodic disturbance more effectively than the other design method of disturbance observers. To compare the effectiveness of the proposed design method in this thesis, the study shows a result that the disturbance observer designed by using a design method of [5] and a proposed method in this thesis estimates the periodic disturbance for a Single-Input/Single-Output stable plant. Next, the study shows that the linear functional disturbance observer for the periodic disturbances designed by using the proposed design method in this thesis could estimate the periodic disturbances for Single-Input/Single-Output unstable plant.

2.8.1 Numerical example 1. A numerical example of disturbance observers for step disturbance for the stable plant

Consider the problem to estimate the periodic disturbance by designing a disturbance observer using a design method in [5] for stable plant $G(s)$ given as

$$G(s) \quad = \quad \frac{s+1}{s^2+5s+6}. \tag{2.49}$$

The period T of the periodic disturbance $d(t)$ is

$$T = \pi. \tag{2.50}$$

The disturbance observer is denoted as

$$\tilde{d}(s) = Q(s)G(s)^{-1}y(s) + Q(s)u(s), \tag{2.51}$$

where $Q(s)$ in (2.51) is the filter satisfying $\lim_{s\to 0} Q(s) = 1$. $Q(s)$ in (2.51) is settled by

$$Q(s) = \frac{1}{(s+1)^2}. \tag{2.52}$$

When the control input $u(t)$ and the periodic output disturbance $d(t)$ are given by

$$u(t) \quad = \quad 0 \tag{2.53}$$

and

$$d(t) = \sum_{i=1}^{5} \sin(it), \tag{2.54}$$

respectively, the response curve of disturbance is estimated by using a design method [5] for the step disturbance. The response curves of disturbance estimations are shown in Figure 2.2. Here, the dotted line shows the periodic output disturbances of $d(t)$ and the solid line shows the disturbance observer of $\tilde{d}(t)$. Figure 2.2 shows that the disturbance observer $\tilde{d}(s)$ in (2.51) for step disturbance could not estimate $\tilde{d}(t)$ effectively.

2.8.2 Numerical example 2. A numerical example of disturbance observers for step disturbance for the stable plant

Consider the problem to obtain the parameterization of all disturbance observers for stable plant $G(s)$ written by

$$G(s) \quad = \quad \frac{s+1}{s^2+5s+6} \tag{2.55}$$

The period T of the periodic disturbance $d(t)$ is

$$T = \pi. \tag{2.56}$$

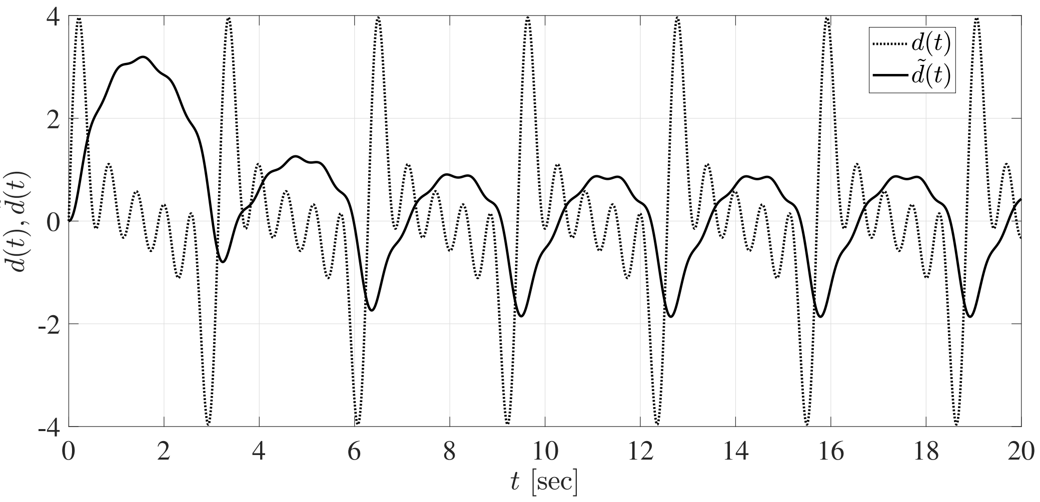

Fig. 2.2: Response curves of the disturbance estimation by using a design method of [5]

Coprime factorization of $G(s)$ in (2.55) satisfying (2.25) is given by

$$\tilde{N}(s) = G(s) = \frac{s+1}{s^2+5s+6} \tag{2.57}$$

and

$$\tilde{D}(s) = \frac{s^2+5s+6}{s^2+13s+42}. \tag{2.58}$$

From Theorem 2.4.1, the parameterization of all disturbance observers $\tilde{d}(s)$ for stable plant $G(s)$ in (2.55) is given by (2.5), where

$$F_1(s) = \frac{s^2+5s+6}{s^2+13s+42} + Q(s)\frac{s^2+5s+6}{s^2+13s+42}, \tag{2.59}$$

$$F_2(s) = -\frac{s+1}{s^2+5s+6} - Q(s)\frac{s+1}{s^2+5s+6} \tag{2.60}$$

and $Q(s) \in RH_\infty$ is any function.

Next using obtained parameterization, we design a disturbance observer $\tilde{d}(s)$ for the periodic output disturbances, that is, $Q(s)$ is settled satisfying (2.26). In order to satisfy (2.26), $Q(s)$ is settled by (2.37).

When the control input $u(t)$ and the periodic output disturbance $d(t)$ are given by

$$u(t) = 0 \tag{2.61}$$

and

$$d(t) = \sum_{i=1}^{5} \sin(it), \tag{2.62}$$

respectively, the response curves of disturbance is estimated by using a proposed method. The response curves of disturbance estimations are shown in Figure 2.3. Here, the dotted line shows the periodic output disturbances of $d(t)$ and the solid line shows the disturbance observer of $\tilde{d}(t)$. Figure 2.3 shows that disturbance observer $\tilde{d}(s)$ in (2.5) for step disturbance could estimate $\tilde{d}(t)$ effectively.

In this way, it is shown that using the obtained parameterization of all disturbance observers for periodic output disturbances, the study could easily design a disturbance observer for step disturbance.

2.8.3 Numerical example 3. A numerical example of disturbance observers for periodic output disturbances

Consider the problem to obtain the parameterization of all disturbance observers for stable plant $G(s)$ written by

$$G(s) = \frac{s+1}{s^2+6s+7} \tag{2.63}$$

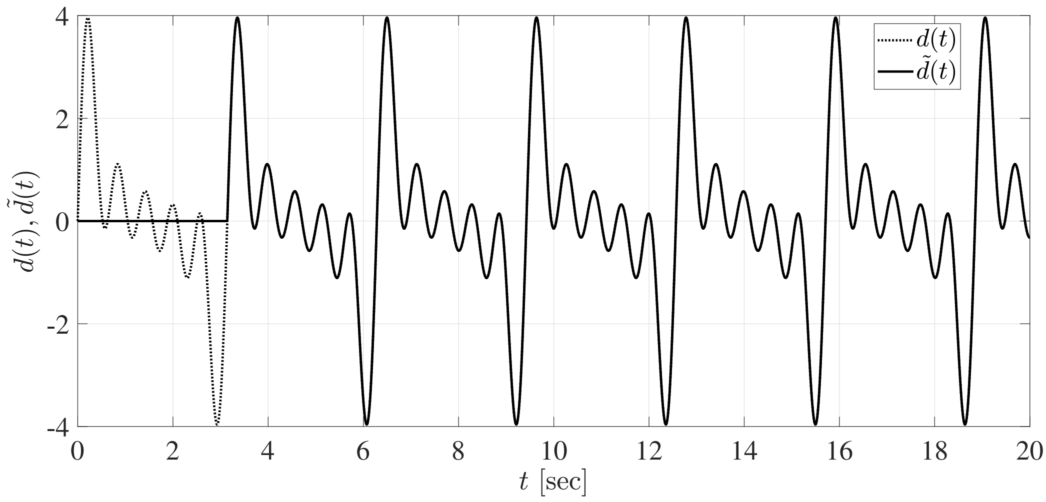

Fig. 2.3: Response curves of the disturbance estimation

The period T of the periodic disturbance $d(t)$ is

$$T = \pi. \tag{2.64}$$

Coprime factorization of $G(s)$ in (2.63) satisfying (2.25) is given by

$$\tilde{N}(s) = G(s) = \frac{s+1}{s^2+6s+7} \tag{2.65}$$

and

$$\tilde{D}(s) = \frac{s^2+6s+7}{s^2+13s+42}. \tag{2.66}$$

From Theorem 2.4.1, the parameterization of all disturbance observers $\tilde{d}(s)$ for stable plant $G(s)$ in (2.63) is given by (2.5), where

$$F_1(s) = \frac{s^2+6s+7}{s^2+13s+42} + Q(s)\frac{s^2+6s+7}{s^2+13s+42}, \tag{2.67}$$

$$F_2(s) = -\frac{s+1}{s^2+6s+7} - Q(s)\frac{s+1}{s^2+6s+7} \tag{2.68}$$

and $Q(s) \in RH_\infty$ is any function.

Next using obtained parameterization, we design a disturbance observer $\tilde{d}(s)$ for the periodic output disturbances, that is, $Q(s)$ is settled satisfying (2.26). In order to satisfy (2.26), $Q(s)$ is settled by (2.37).

When the control input $u(t)$ and the periodic output disturbance $d(t)$ are given by

$$u(t) = 0 \tag{2.69}$$

and

$$d(t) = \begin{cases} 1 + \frac{1}{\pi}t, & 2\pi i \le t < \pi + 2\pi i (\forall i = 0, 1, \ldots) \\ \frac{1}{\pi}t - 1, & \pi + 2\pi i \le t < 2\pi + 2\pi i (\forall i = 0, 1, \ldots) \end{cases}, \tag{2.70}$$

respectively, the response curves of disturbance is estimated by using a proposed method. The response curves of disturbance estimations are shown in Figure 2.4. Here, the dashed line shows the periodic output disturbances of $d(t)$ and the solid line shows the disturbance observer of $\tilde{d}(t)$. Figure 2.4 shows that disturbance observer $\tilde{d}(s)$ in (2.5) for step disturbance could estimate $\tilde{d}(t)$ effectively. The response of the error $e(t)$ in (2.6) is shown in Figure 2.5. Here, the solid line shows the response of $e(t)$. Figure 2.5 shows that disturbance observer $\tilde{d}(s)$ in (2.5) for periodic output disturbances could estimate $d(t) - \tilde{d}(t)$ effectively.

In this way, it is shown that using the obtained parameterization of all disturbance observers for periodic output disturbances, the study could easily design the disturbance observer for periodic output disturbances.

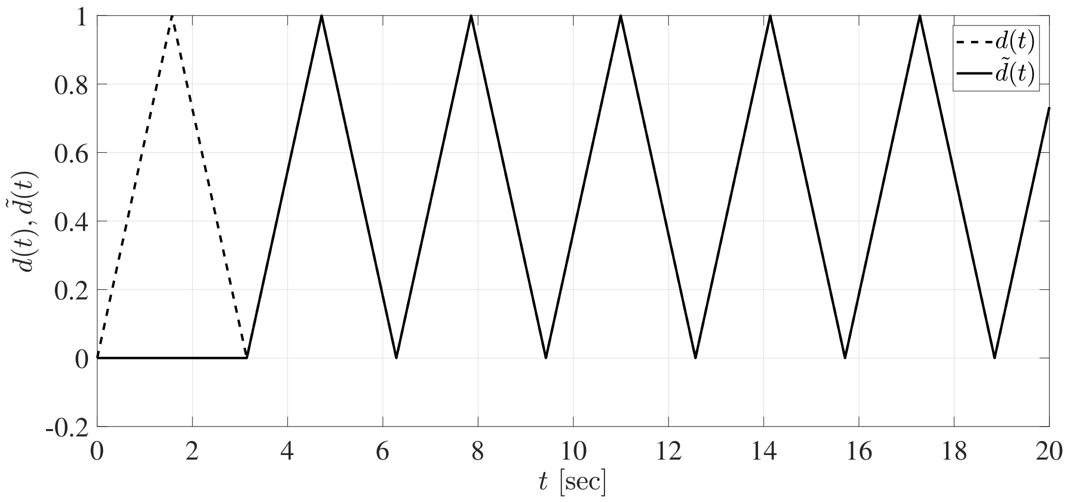

Fig. 2.4: Response curves of the disturbance estimation

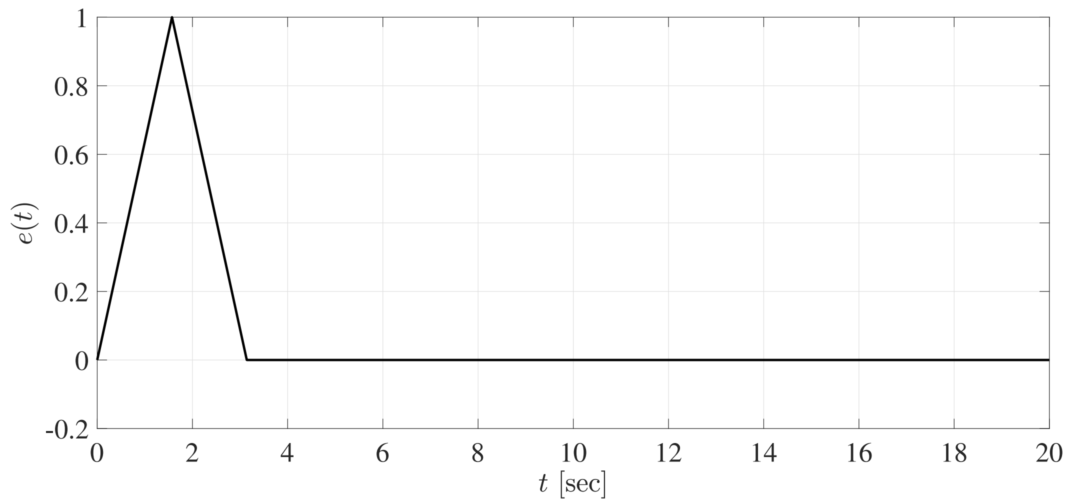

Fig. 2.5: The response of the error $e(t)$ in (2.6)

2.8.4 Numerical example 4. A numerical example for linear functional disturbance observer

Consider the problem to obtain the parameterization of all linear functional disturbance observers for periodic output disturbances for unstable plant $G(s)$ described by

$$G(s) = \frac{s+1}{s^2+43s-350}. \tag{2.71}$$

The period T of the periodic disturbances is

$$T = \pi. \tag{2.72}$$

A pair of coprime factors $\tilde{N}(s) \in RH_\infty$ and $\tilde{D}(s) \in RH_\infty$ of $G(s)$ in (2.71) satisfying (2.25) is given by

$$\tilde{N}(s) = \frac{-2s-2}{s^2+1007s+7000} \tag{2.73}$$

and

$$\tilde{D}(s) = \frac{-2s+100}{s+1000}. \tag{2.74}$$

From Theorem 2.5.1, the parameterization of all linear functional disturbance observers $\tilde{d}(s)$ is given by (2.5), where

$$F_1(s) = \frac{-2s+100}{s+1000} + Q(s)\frac{-2s+100}{s+1000}, \tag{2.75}$$

$$F_2(s) = \frac{2s+2}{s^2+1007s+7000} + Q(s)\frac{2s+2}{s^2+1007s+7000}, \tag{2.76}$$

$$F(s) = 1 - \frac{-2s+100}{s+1000} - Q(s)\frac{-2s+100}{s+1000} \tag{2.77}$$

and $Q(s) \in RH_\infty$ is any function.

Next using obtained parameterization, the study designed a linear functional disturbance observer $\tilde{d}(s)$ for the periodic output disturbances by using the procedure described in Section 2.7, that is, $Q(s)$ is settled satisfying (2.26). The maximum frequency range n_{max} in (2.43) to estimate the periodic disturbance $d(s)$, is settled by

$$n_{max} = 3. \tag{2.78}$$

$\tilde{D}(s)$ in (2.74) is factorized as (2.45), where

$$\tilde{D}_o(s) = \frac{2s+100}{s+1000}, \tag{2.79}$$

and

$$\tilde{D}_i(s) = \frac{-s+50}{s+50}. \tag{2.80}$$

In order to satisfy (2.43), $\hat{Q}(s)$ is settled by

$$\hat{Q}(s) = 1. \tag{2.81}$$

In order to confirm that $\hat{Q}(s)$ in (2.81) satisfy (2.43), the study shows the gain plot of $1 - \hat{Q}(s)\tilde{D}_i(s)$ in Fig. 2.6. Figure 2.6 shows $\hat{Q}(s)$ in (2.81) satisfies (2.43). $Q(s)$ is set by (2.44) and written by

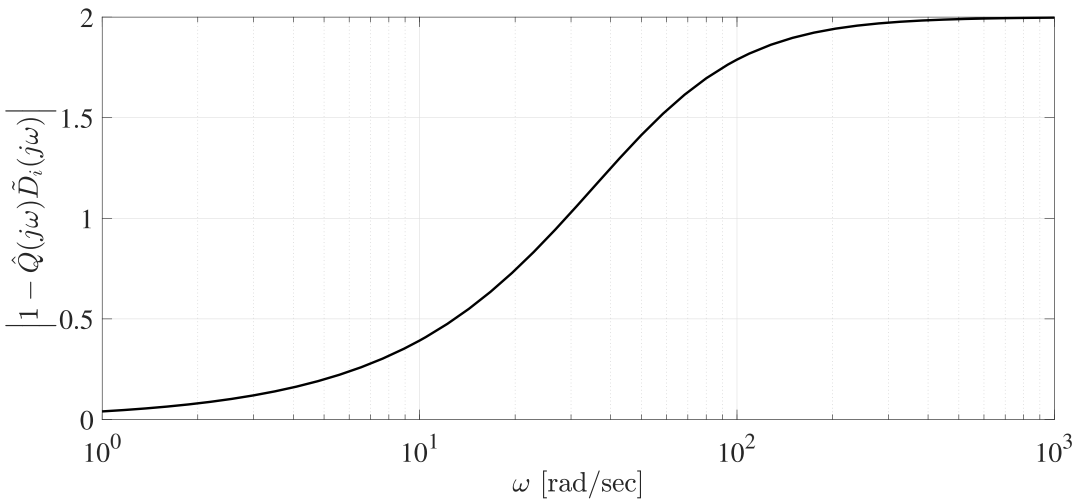

Fig. 2.6: The gain plot of $1 - \hat{Q}(s)\tilde{D}_i(s)$

$$Q(s) = \frac{1.5s+450}{s+50}. \tag{2.82}$$

From (2.40), (2.41) and (2.42), the study has $F_1(s)$, $F_2(s)$ and $F(s)$ designed as

$$F_1(s) = \frac{-5s^2-750s+50000}{s^2+1050s+50000}, \tag{2.83}$$

$$F_2(s) = \frac{5s^2 + 1005s + 1000}{s^3 + 1057s^2 + 57350s + 350000} \tag{2.84}$$

and

$$F(s) = \frac{6s^2 + 1800s}{s^2 + 1050s + 50000}. \tag{2.85}$$

When the control input $u(t)$ and the periodic output disturbance $d(t)$ are given by

$$u(t) = 0 \tag{2.86}$$

and

$$d(t) = \begin{cases} 1 + \frac{1}{\pi}t, & 2\pi i \leq t < \pi + 2\pi i (\forall i = 0, 1, \ldots) \\ \frac{1}{\pi}t - 1, & \pi + 2\pi i \leq t < 2\pi + 2\pi i (\forall i = 0, 1, \ldots) \end{cases}, \tag{2.87}$$

respectively, the response curves of disturbance is estimated by using a proposed method. The response curves of disturbance estimations are shown in Figure 2.7. Here, the dashed line shows the periodic output disturbances of $d(t)$ and the solid line shows the disturbance observer of $\tilde{d}(t)$. Figure 2.7 shows that disturbance observer $\tilde{d}(s)$ in (2.5) for step disturbance could estimate $\tilde{d}(t)$ effectively. The response to the error $e(t)$ in (2.6) is shown

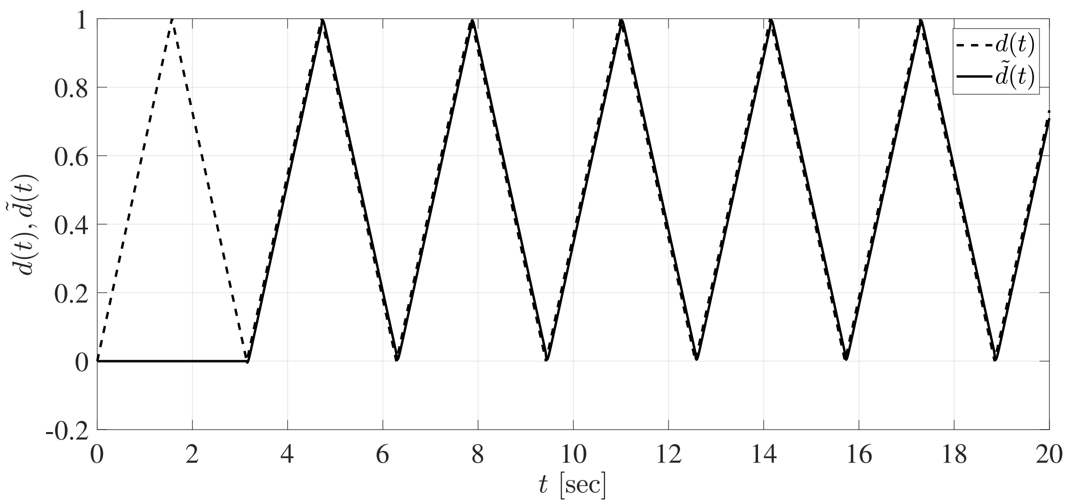

Fig. 2.7: Response curves of the disturbance estimation

in Figure 2.8. Here, the solid line shows the response of $e(t)$. Figure 2.8 shows that linear functional disturbance observer $\tilde{d}(s)$ in (2.5) for periodic output disturbances could estimate $d(t) - \tilde{d}(t)$ effectively.

The study has thus shown that using the parameterization of all linear functional disturbance observers for periodic output disturbances, the study could easily design a linear functional disturbance observer for periodic output disturbances.

2.9 Conclusions

In this chapter, the study has proposed parameterizations of all disturbance observers and of all linear functional disturbance observers for periodic output disturbances. The study shows that the proposed method could attenuate periodic disturbances effectively without using repetitive controllers. A design method and a design procedure of linear functional disturbance observers are presented. Finally, the study shows features of the proposed design method through numerical examples.

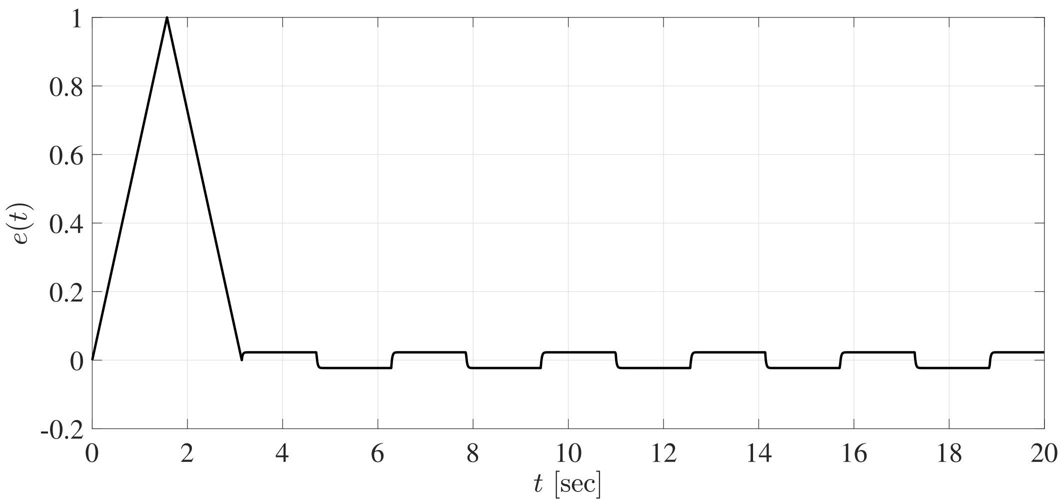

Fig. 2.8: The response of the error $e(t)$ in (2.6)

Chapter 3

Disturbance Observers for Periodic Input Disturbances

3.1 Introduction

In this chapter, the study clarifies that the periodic input disturbances could estimate by using disturbance observers and propose the parameterization of all disturbance observers for periodic input disturbances. First, the necessary structure and characteristics of disturbance observers for periodic input disturbances are defined. In addition, the problem considered in this thesis is explained. Conditions to estimate the periodic input disturbances are clarified. The parameterization of all disturbance observers for periodic input disturbances and that of all linear functional disturbance observers for periodic input disturbances are clarified. In addition, a design method for the linear functional disturbance observer and a procedure for linear functional disturbance observers for periodic input disturbances are clarified. Finally, the study offered a numerical example to illustrate the features of the proposed design method. Fig. 3.1 shows the flowchart for the research process. This chapter is organized comprising: In Section 3.2, the necessary structure and characteristics of disturbance observers for periodic input disturbances are defined. In addition, the problem considered in this thesis is explained. In Section 3.3, conditions to estimate the periodic input disturbances are clarified. In Section 3.4, parameterization of all disturbance observers for periodic disturbances is clarified. In Section 3.5, parameterization of all linear functional disturbance observers for periodic input disturbance is clarified. In Section 3.6, using obtained parameterizations, a design method for the linear functional disturbance observer is presented. In Section 3.7, a design procedure for linear functional disturbance observers for periodic input disturbances is shown. In Section 3.8, the study offer a numerical example to illustrate the features of the proposed design method. In Section 3.9 gives some concluding remarks.

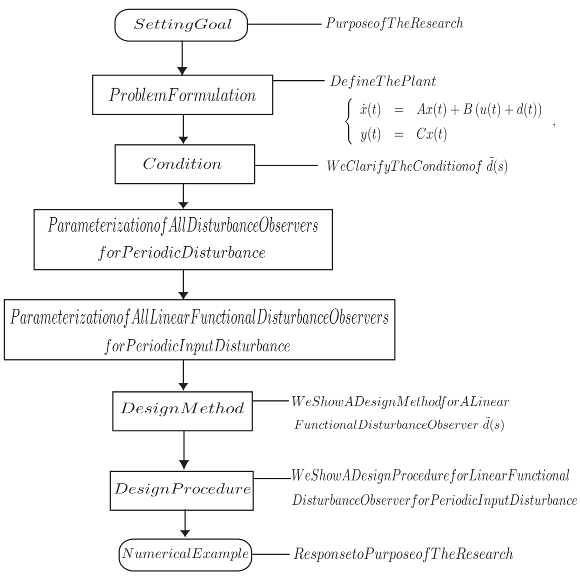

Fig. 3.1: Flowchart for The Research Process.

3.2 Problem Formulation

Consider the plant described by

$$\begin{cases} \dot{x}(t) &= Ax(t) + B\left(u(t) + d(t)\right) \\ y(t) &= Cx(t) \end{cases}, \tag{3.1}$$

where $A \in R^{n\times n}$, $B \in R^{n\times p}$ and $C \in R^{m\times n}$, $x \in R^n$ is the state variable, $u \in R^p$ is the control input, $y \in R^m$ is the output, $d(t) \in R^m$ is periodic disturbances with period $T \geq 0$ satisfying

$$d(t+T) = d(t) \quad (\forall t \geq 0)\,. \tag{3.2}$$

It is assumed that (A, B) is stabilizable, (C, A) is detectable, A has no eigenvalue on the imaginary axis and $u(t)$ and $y(t)$ are available, but $d(t)$ is unavailable. The transfer function from $u(s)$ to $y(s)$ in (3.1) is denoted by

$$y(s) = G(s)u(s) + G(s)d(s), \tag{3.3}$$

where

$$G(s) = C(sI - A)^{-1}B \in R^{m\times p}(s). \tag{3.4}$$

When the disturbances $d(t)$ is unavailable, a disturbance estimator called the disturbance observer is frequently used. The disturbance observer estimates the disturbance of the periodic input disturbance $d(t)$ by using available measurements. Since available measurements of the plant in (3.1) are $u(t)$ and $y(t)$, that is, the input disturbance $d(t)$ satisfies (3.2), the periodic input disturbance $d(s)$ could be estimated by the form in

$$\tilde{d}(s) = F_1(s)e^{-sT}y(s) + F_2(s)e^{-sT}u(s), \tag{3.5}$$

where $F_1(s) \in RH_\infty^{m\times m}$, $F_2(s) \in RH_\infty^{m\times p}$ and $\tilde{d}(t) \in R^m$. The structure of disturbance observer $\tilde{d}(s)$ in (3.5) is shown in Fig. 3.2.

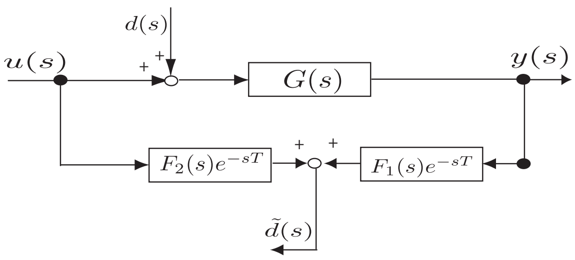

Fig. 3.2: Structure of A Disturbance Observer.

The concept of a disturbance observer for periodic input disturbances is proposed following:

Definition *(disturbance observer for periodic input disturbances)*
The study calls the system $\tilde{d}(s)$ in (3.5) a "disturbance observer for periodic input disturbances", if the error $e(t)$ between $d(t)$ and $\tilde{d}(t)$ written by

$$e(t) = d(t) - \tilde{d}(t) \tag{3.6}$$

satisfies

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty}\left(d(t) - \tilde{d}(t)\right) = 0 \tag{3.7}$$

for any initial state $x(0)$, control input $u(t)$ and periodic disturbances $d(t)$.

If the parameterization of all disturbance observers for periodic input disturbances is obtained, there is a possibility to attenuate periodic input disturbances without using the repetitive control system. In addition, the study can design the disturbance observers for periodic input disturbances systematically. However, no research examines the parameterization of all disturbance observers for periodic input disturbances.

The problem consideration in this thesis is to propose the parameterization of all disturbance observers for periodic input disturbances. Thus, the study obtains the parameterization of all disturbance observers for $\tilde{d}(s)$ in (3.5) for periodic input disturbances

3.3 Condition to Estimate The Periodic Input Disturbances

In this section, the study clarifies the condition of $\tilde{d}(s)$ in (3.5) to satisfy (3.7).

The condition of $\tilde{d}(s)$ in (3.5) satisfying (3.7) is summarized in the following theorem.

Theorem 3.3.1 *$\tilde{d}(s)$ in (3.5) works as a disturbance observer for periodic input disturbances if and only if*

$$F_1(s)N(s) + F_2(s)D(s) = 0. \tag{3.8}$$

$$(1 - \bar{s}_i)^{l_i} \left(I - F_1(\bar{s}_i)e^{-\bar{s}_i T}\right) G(\bar{s}_i) = 0 \ \ (i = 1, \dots, q), \tag{3.9}$$

$$\left(1 - e^{-s_i T}\right) e(s_i) = 0 \ \ \forall s_i (i = 0, 1, \dots) \tag{3.10}$$

are satisfied, where $\bar{s}_i (i = 1, \dots, q)$ is unstable pole of $G(s)$, $l_i (i = 1, \dots, q)$ is its multiplicity, q is the number of unstable poles,

$$s_i \ \ = \ \ j\omega_i, \tag{3.11}$$

$$\omega_i = \frac{2\pi i}{T} \ \ (i = 0, 1, \dots) \tag{3.12}$$

and j is the imaginary unit.

Proof: First the necessity is shown, that is, if $\tilde{d}(s)$ in (3.5) satisfies (3.7), then (3.8) and (3.10) are satisfied. $\tilde{d}(s)$ in (3.5) is rewritten by

$$\begin{aligned}\tilde{d}(s) \ \ &= \ \ (F_1(s)e^{-sT}N(s) + F_2(s)e^{-sT}D(s))\xi(s) \\ &\quad +F_1(s)G(s)e^{-sT}d(s),\end{aligned} \tag{3.13}$$

where $\xi(s)$ is the pseudo-state variable satisfying

$$u(s) = D(s)\xi(s), \tag{3.14}$$

$N(s) \in RH_\infty{}^{m\times p}$ and $D(s) \in RH_\infty{}^{m\times m}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = N(s)D^{-1}(s). \tag{3.15}$$

$\xi(s)$ in (3.13) is factorized as

$$\begin{aligned}\xi(s) \ \ &= \ \ \tilde{\xi}(s) + \bar{\xi}(s) \\ &= \ \ \frac{1}{1 - e^{-sT}}\tilde{\xi}(s) + \bar{\xi}(s),\end{aligned} \tag{3.16}$$

where $\xi(s)$ is denoted by

$$\tilde{\xi}(s) = \int_0^T e^{-sT}\xi(\tau)d\tau, \tag{3.17}$$

$\tilde{\xi}(s)/(1 - e^{-sT})$ means the periodic signal with period T and $\bar{\xi}(s)$ includes all other signals. From (3.2), $d(s)$ in (3.13) is written by

$$d(s) = \frac{1}{1 - e^{-sT}}\hat{d}(s) \tag{3.18}$$

where

$$\hat{d}(s) = \int_0^T e^{-s\tau}d(\tau)d\tau. \tag{3.19}$$

From (3.13) and (3.18), $e(s)$ is given by

$$\begin{aligned}e(s) \ \ &= \ \ \left(I - F_1(s)e^{-sT}\right) G(s)\frac{1}{1 - e^{-sT}}\hat{d}(s) \\ &\quad - \left(F_1(s)N(s) + F_2(s)D(s)\right) \frac{e^{-sT}}{1 - e^{-sT}}\tilde{\xi}(s) \\ &\quad - \left(F_1(s)N(s) + F_2(s)D(s)\right) e^{-sT}\bar{\xi}(s)\end{aligned} \tag{3.20}$$

From the assumption that $e(t)$ satisfies (3.7) for any $\bar{\xi}(s)$,

$$(F_1(s)N(s) + F_2(s)D(s))\, e^{-sT}\bar{\xi}(s) = 0 \tag{3.21}$$

is satisfied for any $\bar{\xi}(s)$. That is, we have (3.8). Substitution of (3.8) to (3.20) gives

$$\begin{aligned} e(s) &= \left(I - F_1(s)e^{-sT}\right) G(s)\frac{1}{1-e^{-sT}}\hat{d}(s) \\ &\quad - (F_1(s)N(s) + F_2(s)D(s))\, \frac{e^{-sT}}{1-e^{-sT}}\tilde{\xi}(s). \end{aligned} \tag{3.22}$$

From the assumption that $e(t)$ satisfies (3.7), $(I - F_1(s)e^{-sT})G(s)$ has no unstable pole. Therefore (3.9) holds true. From the internal model principle [41, 43, 44], (3.10) is satisfied. The study has thus proved the necessity.

Next the sufficiency is shown. That is, if (3.8) and (3.10) are satisfied, then $e(s)$ in (3.6) satisfies (3.7). From (3.8), $e(s)$ in (3.6) is written by

$$\begin{aligned} e(s) &= \left(I - F_1(s)e^{-sT}\right) G(s)\frac{1}{1-e^{-sT}}\hat{d}(s) \\ &\quad - (F_1(s)N(s) + F_2(s)D(s))\, \frac{e^{-sT}}{1-e^{-sT}}\tilde{\xi}(s). \end{aligned} \tag{3.23}$$

From the assumption that (3.7) is for $F_1(s) \in RH_\infty^{m\times m}$ satisfying (3.9). Substituting (3.8) and (3.9) for (3.23), $e(s)$ is rewritten by

$$\begin{aligned} e(\bar{s}_i) &= \left(I - F_1(\bar{s}_i)e^{-\bar{s}_iT}\right) G(\bar{s}_i)\frac{1}{1-e^{-\bar{s}_iT}}\hat{d}(\bar{s}_i) \\ &\quad - (F_1(\bar{s}_i)N(\bar{s}_i) + F_2(\bar{s}_i)D(\bar{s}_i))\, \frac{e^{-\bar{s}_iT}}{1-e^{-\bar{s}_iT}}\tilde{\xi}(\bar{s}_i) \\ &= 0. \end{aligned} \tag{3.24}$$

From (3.10), $e(t)$ in (3.6) satisfies (3.7). Thus the sufficiency is shown.

The study has thus proved Theorem 3.3.1.

Note that from Theorem 3.3.1, when (3.8) is a condition of $\tilde{d}(s)$ disturbance observers for any state variable. In addition, (3.9) and (3.10) are a condition to estimate any periodic signals. Therefore, this is the most important condition to estimate the periodic input disturbances and the mentioned condition could solve the problem.

In this section, the study obtained the conditions to estimate the periodic input disturbances. In the next section, using the result of Theorem 3.3.1, the study clarifies the parameterization of all disturbance observers for periodic input disturbances.

3.4 Parameterization of All Disturbance Observers for Periodic Disturbances

In section, the study proposes the parameterization of all disturbance observer $\tilde{d}\,(s)$ in (3.5) for periodic input disturbance.

The parameterization is summarized in the following theorem.

Theorem 3.4.1 *The system $\tilde{d}(s)$ in (3.5) is the disturbance observer for periodic input disturbance if and only if $F_1(s)$ and $F_2(s)$ are written by*

$$F_1(s) = \tilde{D}(s) + Q(s)\tilde{D}(s), \tag{3.25}$$

$$F_2(s) = -\tilde{N}(s) - Q(s)\tilde{N}(s) \in RH_\infty^{m\times p}, \tag{3.26}$$

and

$$(1-\bar{s}_i)^{l_i}\left(I - F_1(\bar{s}_i)e^{-\bar{s}_iT}\right) G(\bar{s}_i) = 0 \ \ (i = 1, \ldots, q), \tag{3.27}$$

where $\tilde{N}(s) \in RH_\infty^{m\times p}$ *and* $\tilde{D}(s) \in RH_\infty^{m\times m}$ *are coprime factors of* $G(s)$ *on* RH_∞ *satisfying*

$$G(s) = \tilde{D}^{-1}(s)\tilde{N}(s), \tag{3.28}$$

and $Q(s) \in RH_\infty$ *is any function satisfying*

$$\tilde{D}(s_i) + Q(s_i)\tilde{D}(s_i) = I \ \ \forall s_i (i = 0, \ldots), \tag{3.29}$$

satisfying (3.10) and (3.11).

Proof of Theorem 3.4.1 requires following lemma.

Lemma 3.4.1 *[24] Assume that* $A(s) \in RH_\infty^{m\times n}$*,* $B(s) \in RH_\infty^{q\times p}$*,* $C(s) \in RH_\infty^{m\times p}$ *and*

$$\text{rank} \left[\begin{array}{cc} A^T(s) & B^T(s) \end{array} \right] = \gamma. \tag{3.30}$$

are satisfied. There exist $X(s) \in RH_\infty$ *and* $Y(s) \in RH_\infty$ *satisfying*

$$X(s)A(s) + Y(s)B(s) = C(s) \tag{3.31}$$

if and only if there exists $U(s) \in \mathcal{U}$ *satisfying*

$$\left[\begin{array}{c} A(s) \\ B(s) \\ C(s) \end{array} \right] = U(s) \left[\begin{array}{c} A(s) \\ B(s) \\ 0 \end{array} \right]. \tag{3.32}$$

When $X_0(s) \in RH_\infty$ *and* $Y_0(s) \in RH_\infty$ *are solution to (3.31), then all solutions to (3.31) are given by*

$$\begin{aligned} \left[\begin{array}{cc} X(s) & Y(s) \end{array} \right] &= \left[\begin{array}{cc} X_0(s) & Y_0(s) \end{array} \right] \\ &\quad + Q(s) \left[\begin{array}{cc} W_1(s) & W_2(s) \end{array} \right], \end{aligned} \tag{3.33}$$

where $W_1(s)$ *and* $W_2(s)$ *satisfy*

$$W_1(s)A(s) + W_2(s)B(s) = 0 \tag{3.34}$$

and

$$\text{rank} \left[\begin{array}{cc} W_1(s) & W_2(s) \end{array} \right] = n + q - \gamma \tag{3.35}$$

and $Q(s) \in RH_\infty^{p\times(N+q-\gamma)}$ *is any function.*

Using Theorem 3.3.1 and Lemma 3.4.1, Theorem 3.4.1 is proved.
Proof: From Theorem 3.3.1, $\tilde{d}(s)$ works a disturbance observer for periodic disturbance if and only if $F_1(s) \in RH_\infty^{m\times m}$ and $F_2(s) \in RH_\infty^{m\times p}$ satisfy (3.8). From Lemma 3.4.1, all solution of $F_1(s)$ and $F_2(s)$ to satisfy (3.8) are given by (3.25) and (3.26), respectively, since

$$\tilde{D}(s)N(s) - \tilde{N}(s)D(s) = 0, \tag{3.36}$$

and Lemma 3.4.1, where $\tilde{D}(s) \in RH_\infty^{p\times p}$ and $\tilde{N}(s) \in RH_\infty^{p\times m}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \tilde{D}^{-1}(s)\tilde{N}(s). \tag{3.37}$$

The rest is to prove $\tilde{d}(s)$ in (3.5) works as a periodic input disturbance observer if and only if $Q(s)$ in (3.25) and (3.26) satisfy (3.29). From Theorem 3.3.1, $\tilde{d}(s)$ in (3.5) works as a periodic input disturbance observer if and only if $e(s)$ in (3.6) satisfy (3.10). The necessity is shown. That is if $\tilde{d}(s)$ in (3.5) works as a periodic input disturbance observer, then $Q(s)$ in (3.25) and (3.26) satisfy (3.29). From (3.25) and (3.26), $e(s)$ in (3.6) is written by

$$e(s) = \left\{ I - F_1(s)e^{-sT} \right\} G(s) \frac{1}{1 - e^{-sT}} \hat{d}(s). \tag{3.38}$$

From the assumption that (3.7) is for $F_1(s) \in RH_\infty^{m\times m}$ satisfying (3.27). Substituting (3.27) for (3.38), $e(s)$ is rewritten by

$$\left(1 - e^{-\bar{s}_i T}\right) e(\bar{s}_i) = \left\{ I - F_1(\bar{s}_i)e^{-\bar{s}_i T} \right\} G(\bar{s}_i)\tilde{d}(\bar{s}_i) \tag{3.39}$$

This equation yields

$$\begin{aligned}\left(1-e^{-\bar{s}_i T}\right) e(\bar{s}_i) &= \left\{I-\left(\tilde{D}(\bar{s}_i)+Q(\bar{s}_i)\tilde{D}(\bar{s}_i)\right)\right\} G(\bar{s})\tilde{d}(\bar{s}_i) \\ &= 0. \end{aligned} \quad (3.40)$$

The study has (3.29). Thus the study has proved the necessity.

Next, the sufficiency is shown. That is, the study shows that if $Q(s)$ in (3.25) and (3.26) satisfy (3.29), then (3.10) is satisfied. $e(s)$ in (3.6) is written by (3.38). Substituting (3.29) to (3.38), it is obvious that (3.10) is satisfied. In this way, the sufficiency has been proved.

From the above discussion, the study has thus proved Theorem 3.4.1.

Note that from Theorem 3.4.1, when $G(s)$ is stable, if $Q(s)$ is settled by

$$Q(s) = \tilde{D}^{-1}(s) - I, \quad (3.41)$$

then $Q(s)$ in (3.41) satisfies (3.29). However when $G(s)$ is unstable, it is difficult to set $Q(s)$ satisfying (3.29). For the unstable plant $G(s)$, a disturbance observer for periodic input disturbances is often used to attenuate disturbances effectively in [42], even if the system $\tilde{d}(s)$ in (3.5) satisfying (3.10) could not be designed. This means that in order to attenuate periodic disturbances, it is enough to estimate $(I - F(s))G(s)\tilde{d}(s)$, where $F(s) \in RH_\infty$ is any function. From this point of view, in the next section, when $G(s)$ is unstable, the study defines a linear functional disturbance observer for periodic input disturbance observer and clarify the parameterization of all linear functional disturbance observers for periodic input observers.

3.5 Parameterization of All Linear Functional Disturbance Observers for Periodic Input Disturbances

In this section, the study defines a linear functional disturbance observer and presents the parameterization of all linear functional disturbance observers for periodic input disturbance.

The study calls $\tilde{d}(s)$ in (3.5) the linear functional disturbance observer for periodic input disturbances if $\tilde{d}(s)$ is written by

$$\left(1-e^{-s_i T}\right) e(s_i) = F(s_i)G(s_i)\hat{d}(s_i) \quad (3.42)$$

is satisfied, where $F(s) \in RH_\infty$ is any function satisfying

$$\bar{\sigma}\left\{F(s_i)\right\} \simeq 0 \ \ \forall s_i (i = 1, \ldots, n_{max}) \quad (3.43)$$

and n_{max} is the maximum frequency satisfying (3.43). Since the available measurements of the plant $G(s)$ in (3.1) are $u(t)$ and $y(t)$ and the input disturbance $d(t)$ satisfies (3.2), the periodic disturbance d(t) is estimated by the form in (3.5), where $F_1(s) \in RH_\infty^{m\times m}$ and $F_2(s) \in RH_\infty^{m\times p}$.

The parameterization of the linear functional disturbance observer for periodic input disturbance is summarized following.

Theorem 3.5.1 *The system $\tilde{d}(s)$ in (3.5) is the linear functional disturbance observer for periodic input disturbance if and only if $F_1(s)$, $F_2(s)$ and $F(s)$ are described by*

$$F_1(s) = \tilde{D}(s) + Q(s)\tilde{D}(s), \quad (3.44)$$

$$F_2(s) = -\tilde{N}(s) - Q(s)\tilde{N}(s), \quad (3.45)$$

and

$$\begin{aligned} F(s) &= I - F_1(s) \\ &= I - \left(\tilde{D}(s) + Q(s)\tilde{D}(s)\right), \end{aligned} \quad (3.46)$$

respectively, where $Q(s)$ is any function satisfying

$$\begin{aligned}(1-\bar{s}_i)^{l_i}\left(I - F_1(\bar{s}_i)e^{-\bar{s}_i T}\right)G(\bar{s}_i) &= (1-\bar{s}_i)^{l_i}\left\{I-\left(\tilde{D}(\bar{s}_i)+Q(\bar{s}_i)\tilde{D}(\bar{s}_i)e^{-\bar{s}_i T}\right)\right\}G(\bar{s}_i) \\ &= 0 \ \ (i = 1, \ldots, q) \end{aligned} \quad (3.47)$$

and

$$\begin{aligned}\bar{\sigma}\left(I - F_1(s_i)\right) &= \bar{\sigma}\left\{I-\left(\tilde{D}(s_i)+Q(s_i)\tilde{D}(s_i)\right)\right\} \\ &\simeq 0 \ \ \forall s_i (i = 1, \ldots, n_{max}). \end{aligned} \quad (3.48)$$

Proof: First, the necessity is shown. That is, the study shows that if the system $\tilde{d}(s)$ in (3.5) is a linear functional disturbance observer for a periodic input disturbances, then (3.44), (3.45), (3.46), (3.47) and (3.48) are satisfied. From (3.5), (3.13), (3.14), (3.15), (3.16) and (3.17), for the system $\tilde{d}(s)$ in (3.5), $e(s)$ is written as (3.20). From the assumption that $e(s)$ satisfies (3.7) for any $\bar{\xi}(s)$, (3.22) holds for any $\bar{\xi}(s)$. That is, the study has (3.10). From (3.36) and Lemma 3.4.1, all solutions of $F_1(s)$ and $F_2(s)$ to satisfy (3.10) are given by (3.44) and (3.45), respectively. Substitution of (3.10) to (3.22) gives (3.23). From (3.23) the assumption that $e(s)$ satisfies (3.42), the study has (3.46), (3.47) and (3.48). In this way, the necessity has been proved.

Next, the sufficiency is shown. That is, the study shows that if (3.44), (3.45), (3.46), (3.47) and (3.48) is satisfied, then the $\tilde{d}(s)$ is a linear functional disturbance observer. Since $e(s)$ in (3.6) is written by (3.20), Substituting (3.44), (3.45), (3.46), (3.47) and (3.48) to (3.20), it is obvious that (3.42) is satisfied. In this way, the sufficiency has been proved.

From the above, the study has thus proved Theorem 3.5.1.
Note that from Theorem 3.5.1, $\tilde{d}(s)$ satisfying (3.42) and (3.43), then (3.44), (3.45), (3.46), (3.47) and (3.48) are satisfied to be solved by Theorem 3.5.1.

3.6 Design Method for Linear Functional Disturbance Observers

In this section, the study shows a design method for a linear functional disturbance observer $\tilde{d}(s)$.

In order to design the linear functional disturbance observer $\tilde{d}(s)$ for periodic input disturbances, $Q(s)$ in (3.44) and (3.45) needs to satisfy (3.48).

When $G(s)$ is unstable, $Q(s)$ is set as

$$Q(s) = \hat{Q}(s)\left(I - \tilde{D}(s)\right)\tilde{D}_o^{-1}(s), \tag{3.49}$$

and

$$(1-\bar{s}_i)^{l_i}\left(I - (\tilde{D}(\bar{s}_i) + Q(\bar{s}_i)\tilde{D}(\bar{s}_i))e^{-\bar{s}_i T}\right)G(\bar{s}_i) = 0 \quad (i = 1, \ldots, q), \tag{3.50}$$

where $\tilde{D}_o(s) \in RH_\infty^{m\times m}$ is an outer function of $\tilde{D}(s)$ satisfying

$$\tilde{D}(s) = \tilde{D}_o(s)\tilde{D}_i(s), \tag{3.51}$$

$\tilde{D}_i(s) \in RH_\infty^{m\times m}$ is a co-inner function of $\tilde{D}(s)$ satisfying $\tilde{D}_i(0) = I$ and $\tilde{D}_i(s)\tilde{D}_i(-s)^T = I$, $\hat{Q}(s) \in RH_\infty^{m\times m}$ is any function satisfying

$$(1-\bar{s}_i)^{l_i}\left\{I - \hat{Q}(\bar{s}_i)\tilde{D}_i(\bar{s}_i)e^{-\bar{s}_i T}\right\}G(\bar{s}_i) = 0 \quad (i = 1, \ldots, q) \tag{3.52}$$

and

$$\bar{\sigma}\left\{I - \hat{Q}(\bar{s}_i)\tilde{D}_i(\bar{s}_i)\right\} \simeq 0 \quad \forall s_i (i = 1, \ldots, n_{max}). \tag{3.53}$$

From the above, the study showed a design of the linear functional disturbance observer $\tilde{d}(s)$ for periodic input disturbances, $Q(s)$ in (3.49) and (3.50) satisfied (3.51), (3.52) and (3.53) based on Theorem 3.5.1. When $Q(s)$ in (3.49) and (3.50) are designed using the method described In Section 3.7

3.7 Design Procedure for Linear Functional Disturbance Observers for Periodic Input Disturbance

In this section, the study shows a design procedure for linear functional disturbance observer for periodic input disturbance satisfying Theorem 3.5.1.

A design procedure is summarized following:

<u>Procedure</u>

Step 1) Obtain coprime factors $\tilde{N}(s) \in RH_\infty^{m\times p}$ and $\tilde{D}(s) \in RH_\infty^{m\times m}$ of $G(s) \in R(s)^{m\times p}$ satisfying (3.28). The parameterization of all linear functional disturbance observers is given by (3.5), where $F_1(s)$, $F_2(s)$ and $F(s)$ are written by (3.44), (3.45) and (3.46), respectively.

Step 2) The maximum frequency range n_{max} in (3.48) to estimate the periodic disturbance $d(s)$ is settled.

Step 3) Factorize $\tilde{D}(s)$ as (3.51) satisfying $\tilde{D}_i(0) = I$.

Step 4) Settle $Q(s) \in RH_\infty^{m\times m}$ satisfying (3.48). In order to satisfy (3.48), $Q(s) \in RH_\infty^{m\times m}$ is set according to (3.49). Where $\hat{Q}(s)$ is a low-pass filter satisfying $\hat{Q}(0) = I$, as

$$\hat{Q}(s) = \mathrm{diag}\left\{\frac{k_1}{(1+s\tau_1)^{\alpha_1}}, \ldots, \frac{k_m}{(1+s\tau_m)^{\alpha_m}}\right\}, \tag{3.54}$$

$\alpha_i (i = 1, 2, \ldots, m)$ is an arbitrary positive integer and $k_i (i = 1, 2, \ldots, m)$ satisfying $\tau_i (i = 1, \ldots, m)$

$$(1-\bar{s}_i)^{l_i}\left\{I - \hat{Q}(\bar{s}_i)\tilde{D}_i(\bar{s}_i)e^{-\bar{s}_i T}\right\} G(\bar{s}_i) = 0 \quad (i = 1, \ldots, m) \tag{3.55}$$

and

$$\bar{\sigma}\left\{I - \hat{Q}(\bar{s}_i)\tilde{D}_i(\bar{s}_i)\right\} \simeq 0 \quad \forall s_i (i = 1, \ldots, m). \tag{3.56}$$

are real numbers.

Step 5) Substituting $Q(s)$ for (3.44), (3.45) and (3.46), $F_1(s)$, $F_2(s)$ and $F(s)$ are obtained. Then the study could design disturbance observer $\tilde{d}(s)$ for periodic input disturbances as (3.5).

3.8 Numerical Example

In this section, the study shows numerical examples to illustrate the effective of the proposed parameterizations.

Firstly, the study shows that the proposed design method of the disturbance observer for the stable plant in this paper could estimate the periodic disturbance more effective than the other design method of disturbance observers. To compare the effectiveness of the proposed design method in this thesis, the study shows a result that the disturbance observer designed by using a design method of [5] and a proposed method in this thesis estimates the periodic disturbance for a Single-Input/Single-Output stable plant. Next, the study shows that the linear functional disturbance observer for the periodic disturbances designed by using the proposed design method in this thesis could estimate the periodic disturbances for Single-Input/Single-Output unstable plant.

3.8.1 Numerical example 1. A numerical example of disturbance observers for step disturbance for the stable plant

Consider the problem to estimate the periodic disturbance by designing a disturbance observer using a design method in [5] for stable plant $G(s)$ given as

$$G(s) = \frac{s+1}{s^2+4s+5}. \tag{3.57}$$

The period T of the periodic disturbance $d(t)$ is

$$T = \pi. \tag{3.58}$$

The disturbance observer is denoted as

$$\tilde{d}(s) = Q(s)G(s)^{-1}y(s) + Q(s)u(s), \tag{3.59}$$

where $Q(s)$ in (3.59) is the filter satisfying $\lim_{s\to 0} Q(s) = 1$. $Q(s)$ in (3.59) is settled by

$$Q(s) = \frac{1}{(s+1)^2}. \tag{3.60}$$

When the control input $u(t)$ and the periodic input disturbance $d(t)$ are given by

$$u(t) = 0 \tag{3.61}$$

and

$$d(t) = \sum_{i=1}^{3} \sin(it), \tag{3.62}$$

respectively, the response curve of disturbance is estimated by using a design method [5] for the step disturbance. The response curves of disturbance estimations are shown in Figure 3.3. Here, the dotted line shows the periodic input disturbances of $d(t)$ and the solid line shows the disturbance observer of $\tilde{d}(t)$. Figure 3.3 shows that the disturbance observer $\tilde{d}(s)$ in (3.59) for step disturbance could not estimate $\tilde{d}(t)$ effectively.

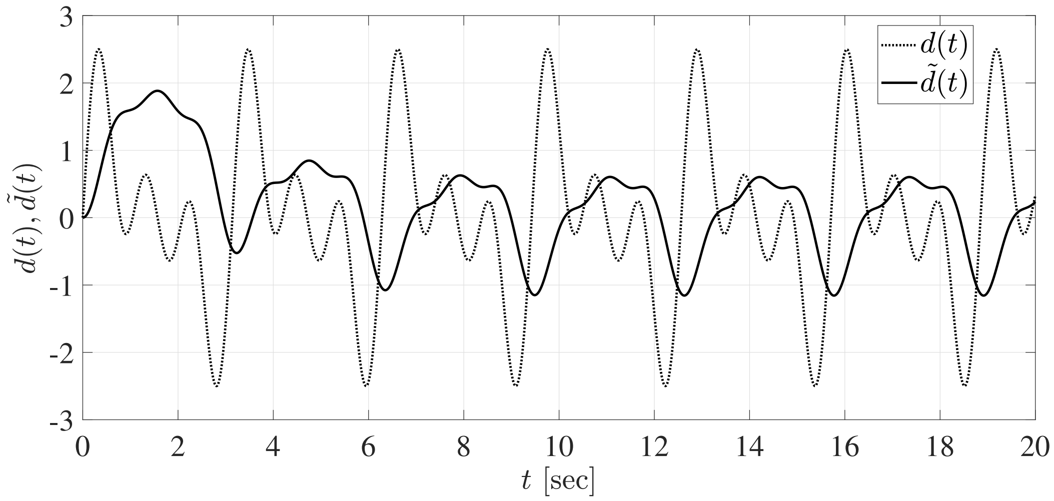

Fig. 3.3: Response Curves of The Disturbance Estimation by Using A Design Method of [5]

3.8.2 Numerical example 2. A numerical example of disturbance observers for step disturbance for the stable plant

Consider the problem to obtain the parameterization of all disturbance observers for stable plant $G(s)$ written by

$$G(s) = \frac{s+1}{s^2+4s+5} \tag{3.63}$$

The period T of the periodic disturbance $d(t)$ is

$$T = \pi. \tag{3.64}$$

Coprime factorization of $G(s)$ in (3.63) satisfying (3.28) is given by

$$\tilde{N}(s) = G(s) = \frac{s+1}{s^2+4s+5} \tag{3.65}$$

and

$$\tilde{D}(s) = \frac{s^2+4s+5}{s^2+13s+42}. \tag{3.66}$$

From Theorem 3.4.1, the parameterization of all disturbance observers $\tilde{d}(s)$ for stable plant $G(s)$ in (3.63) is given by (3.5), where

$$F_1(s) = \frac{s^2+4s+5}{s^2+13s+42} + Q(s)\frac{s^2+4s+5}{s^2+13s+42}, \tag{3.67}$$

$$F_2(s) = -\frac{s+1}{s^2+4s+5} - Q(s)\frac{s+1}{s^2+4s+5} \tag{3.68}$$

and $Q(s) \in RH_\infty$ is any function.

Next using obtained parameterization, the study design a disturbance observer $\tilde{d}(s)$ for the periodic input disturbances, that is, $Q(s)$ is settled satisfying (3.29). In order to satisfy (3.29), $Q(s)$ is settled by (3.41).

When the control input $u(t)$ and the periodic input disturbance $d(t)$ are given by

$$u(t) = 0 \tag{3.69}$$

and

$$d(t) = \sum_{i=1}^{3} \sin(it), \tag{3.70}$$

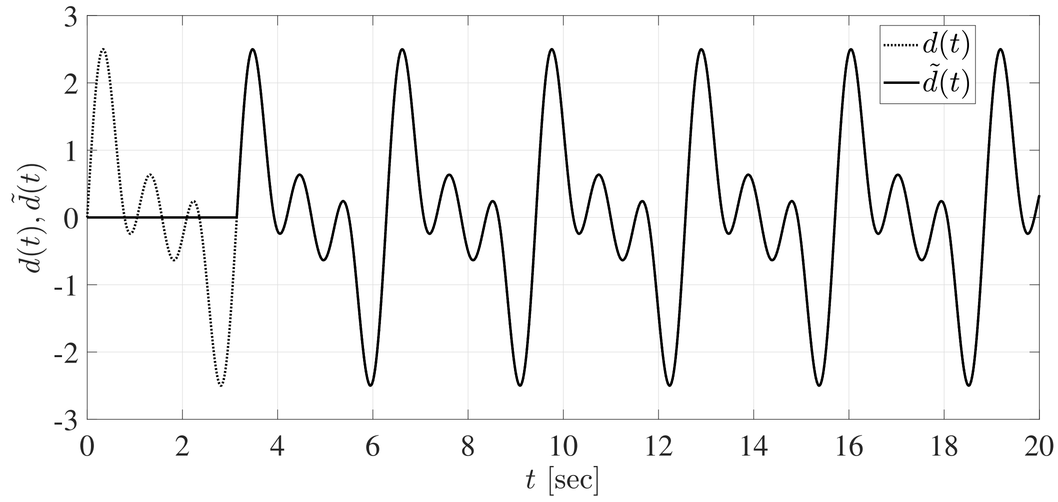

Fig. 3.4: Response Curves of The Disturbance Estimation

respectively, the response curves of disturbance is estimated by using a proposed method. The response curves of disturbance estimations are shown in Figure 3.4. Here, the dotted line shows the periodic input disturbances of $d(t)$ and the solid line shows the disturbance observer of $\tilde{d}(t)$. Figure 3.4 shows that disturbance observer $\tilde{d}(s)$ in (3.5) for step disturbance could estimate $\tilde{d}(t)$ effectively.

In this way, it is shown that using the obtained parameterization of all disturbance observers for periodic input disturbances, the study could easily design a disturbance observer for step disturbance.

3.8.3 Numerical example 3. A numerical example of disturbance observers for periodic input disturbances

Consider the problem to obtain the parameterization of all disturbance observers for stable plant $G(s)$ written by

$$G(s) = \frac{s+1}{s^2+2s+3} \tag{3.71}$$

The period T of the periodic disturbance $d(t)$ is

$$T = \pi. \tag{3.72}$$

Coprime factorization of $G(s)$ in (3.71) satisfying (3.28) is given by

$$\tilde{N}(s) = G(s) = \frac{s+1}{s^2+2s+3} \tag{3.73}$$

and

$$\tilde{D}(s) = \frac{s^2+2s+3}{s^2+13s+42}. \tag{3.74}$$

From Theorem 3.4.1, the parameterization of all disturbance observers $\tilde{d}(s)$ for stable plant $G(s)$ in (3.71) is given by (3.5), where

$$F_1(s) = \frac{s^2+2s+3}{s^2+13s+42} + Q(s)\frac{s^2+2s+3}{s^2+13s+42}, \tag{3.75}$$

$$F_2(s) = -\frac{s+1}{s^2+2s+3} - Q(s)\frac{s+1}{s^2+2s+3} \tag{3.76}$$

and $Q(s) \in RH_\infty$ is any function.

Next using obtained parameterization, the study design a disturbance observer $\tilde{d}(s)$ for the periodic input disturbances, that is, $Q(s)$ is settled satisfying (3.29). In order to satisfy (3.29), $Q(s)$ is settled by (3.41).

When the control input $u(t)$ and the periodic input disturbance $d(t)$ are given by

$$u\,(t) \quad = \quad 0 \tag{3.77}$$

and

$$d\,(t) = \begin{cases} 1 + \dfrac{1}{\pi}t, & 2\pi i \leq t < \pi + 2\pi i (\forall i = 0, 1, \ldots) \\ \frac{1}{\pi}t - 1, & \pi + 2\pi i \leq t < 2\pi + 2\pi i (\forall i = 0, 1, \ldots) \end{cases},$$

respectively, the response curves of disturbance is estimated by using a proposed method. The response curves of disturbance estimations are shown in Figure 3.5. Here, the dashed line shows the periodic input disturbances of $d(t)$ and the solid line shows the disturbance observer of $\tilde{d}(t)$. Figure 3.5 shows that disturbance observer $\tilde{d}\,(s)$ in (3.5) for step disturbance could estimate $\tilde{d}(t)$ effectively. The response of the error $e(t)$ in (3.6) is shown

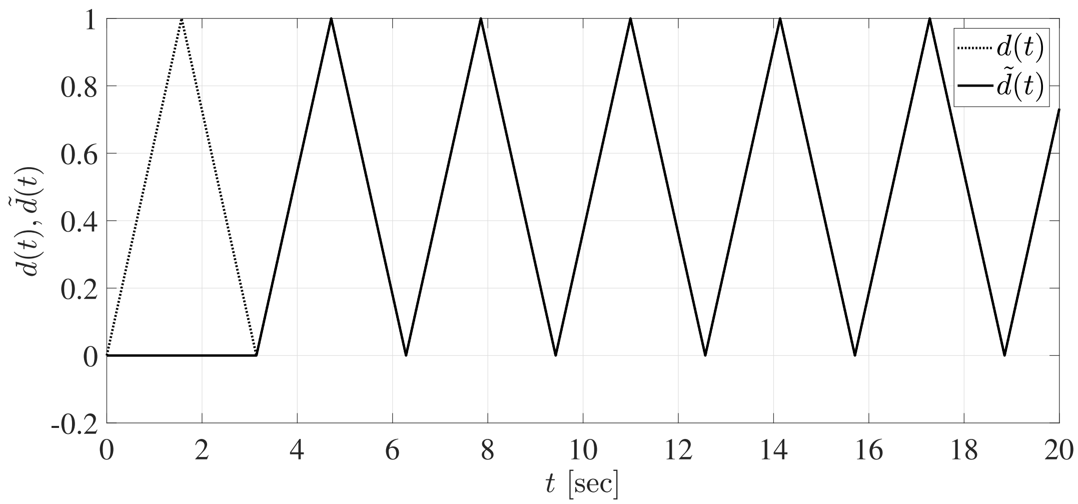

Fig. 3.5: Response Curves of The Disturbance Estimation

in Figure 3.6. Here, the solid line shows the response of $e\,(t)$. Figure 3.6 shows that disturbance observer $\tilde{d}\,(s)$

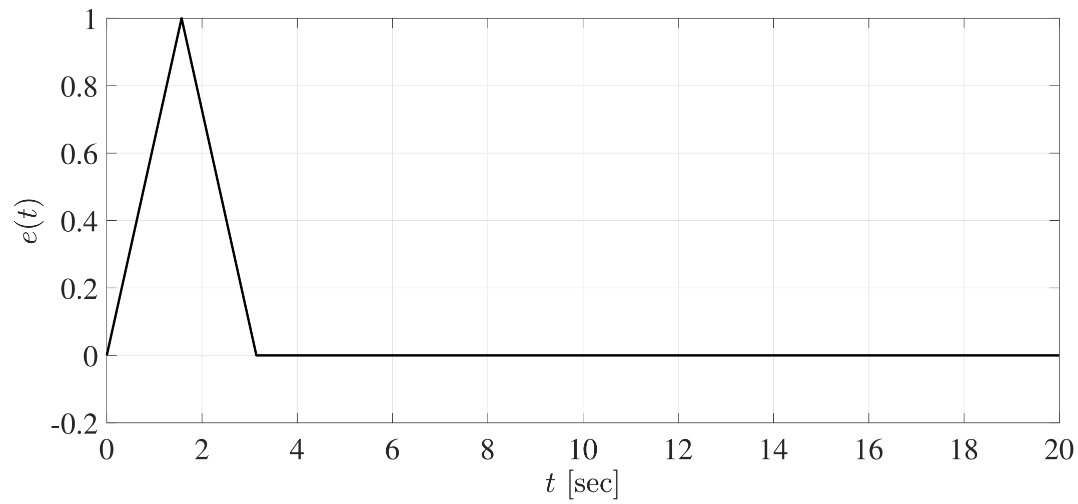

Fig. 3.6: The Response of The Error $e(t)$ in (3.6)

in (3.5) for periodic input disturbances could estimate $d(t) - \tilde{d}(t)$ effectively.

In this way, it is shown that using the obtained parameterization of all disturbance observers for periodic input disturbances, the study could easily design the disturbance observer for periodic input disturbances.

3.8.4 Numerical example 4. A numerical example for linear functional disturbance observer

Consider the problem to obtain the parameterization of all linear functional disturbance observers for periodic input disturbances for unstable plant $G(s)$ described by

$$G(s) = \frac{s+1}{s^2-70s-624}. \tag{3.78}$$

The period T of the periodic disturbances is

$$T = \pi. \tag{3.79}$$

A pair of coprime factors $\tilde{N}(s) \in RH_\infty$ and $\tilde{D}(s) \in RH_\infty$ of $G(s)$ in (3.78) satisfying (3.28) is given by

$$\tilde{N}(s) = \frac{-2s-2}{s^2+1008s+8000} \tag{3.80}$$

and

$$\tilde{D}(s) = \frac{-2s+156}{s+1000}. \tag{3.81}$$

From Theorem 3.5.1, the parameterization of all linear functional disturbance observers $\tilde{d}(s)$ is given by (3.5), where

$$F_1(s) = \frac{-2s+156}{s+1000} + Q(s)\frac{-2s+156}{s+1000}, \tag{3.82}$$

$$F_2(s) = \frac{2s+2}{s^2+1008s+8000} + Q(s)\frac{2s+2}{s^2+1008s+8000}, \tag{3.83}$$

$$F(s) = 1 - \frac{-2s+156}{s+1000} - Q(s)\frac{-2s+156}{s+1000} \tag{3.84}$$

and $Q(s) \in RH_\infty$ is any function.

Next using obtained parameterization, the study design a linear functional disturbance observer $\tilde{d}(s)$ for the periodic input disturbances by using the procedure described in Section 3.7, that is, $Q(s)$ is settled satisfying (3.29). The maximum frequency range n_{max} in (3.48) to estimate the periodic disturbance $d(s)$, is settled by

$$n_{max} = 3. \tag{3.85}$$

$\tilde{D}(s)$ in (3.81) is factorized as (3.51), where

$$\tilde{D}_o(s) = \frac{2s+156}{s+1000}, \tag{3.86}$$

and

$$\tilde{D}_i(s) = \frac{-s+78}{s+78}. \tag{3.87}$$

In order to satisfy (3.48), $\hat{Q}(s)$ is settled by

$$\hat{Q}(s) = 1. \tag{3.88}$$

In order to confirm that $\hat{Q}(s)$ in (3.88) satisfy (3.48), the study shows the gain plot of $1 - \hat{Q}(s)\tilde{D}_i(s)$ in Fig. 3.7. Figure 3.7 shows $\hat{Q}(s)$ in (3.88) satisfies (3.48). $Q(s)$ is set by (3.49) and written by

$$Q(s) = \frac{1.5s+422}{s+78}. \tag{3.89}$$

From (3.44), (3.45) and (3.46), we have $F_1(s)$, $F_2(s)$ and $F(s)$ are designed as

$$F_1(s) = \frac{-5s^2-610s+78000}{s^2+1078s+78000}, \tag{3.90}$$

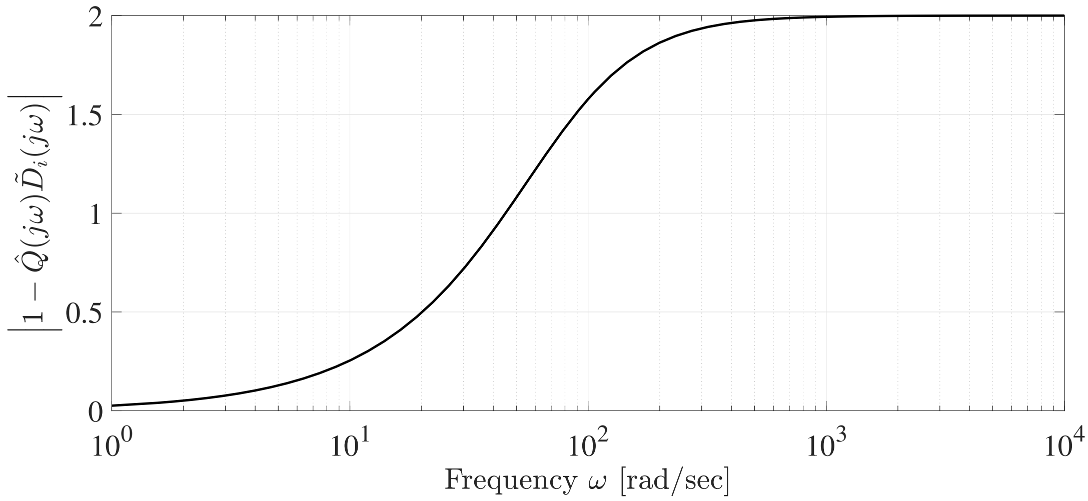

Fig. 3.7: The Gain Plot of $1-\hat{Q}(s)\tilde{D}_i(s)$

$$F_2(s) = \frac{-5s^2-1005s+1000}{s^3+1068s^2+86620s+624000} \tag{3.91}$$

and

$$F(s) = \frac{6s^2+1688s}{s^2+1078s+78000}. \tag{3.92}$$

When the control input $u(t)$ and the periodic input disturbance $d(t)$ are given by

$$u(t)=0 \tag{3.93}$$

and

$$d(t)=\begin{cases} 1+\frac{1}{\pi}t, & 2\pi i \le t < \pi+2\pi i(\forall i=0,1,\ldots) \\ \frac{1}{\pi}t-1, & \pi+2\pi i \le t < 2\pi+2\pi i(\forall i=0,1,\ldots) \end{cases},$$

respectively, the response curves of disturbance is estimated by using a proposed method. The response curves of disturbance estimations are shown in Figure 3.8. Here, the dashed line shows the periodic input disturbances of $d(t)$ and the solid line shows the disturbance observer of $\tilde{d}(t)$. Figure 3.8 shows that disturbance observer $\tilde{d}(s)$ in (3.5) for step disturbance could estimate $\tilde{d}(t)$ effectively. The response to the error $e(t)$ in (3.6) is shown

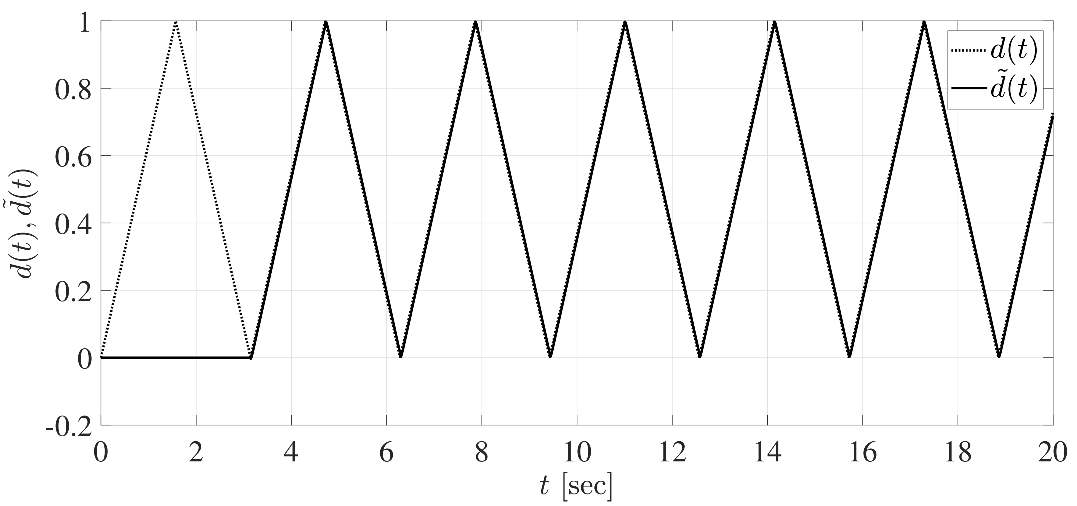

Fig. 3.8: Response Curves of The Disturbance Estimation

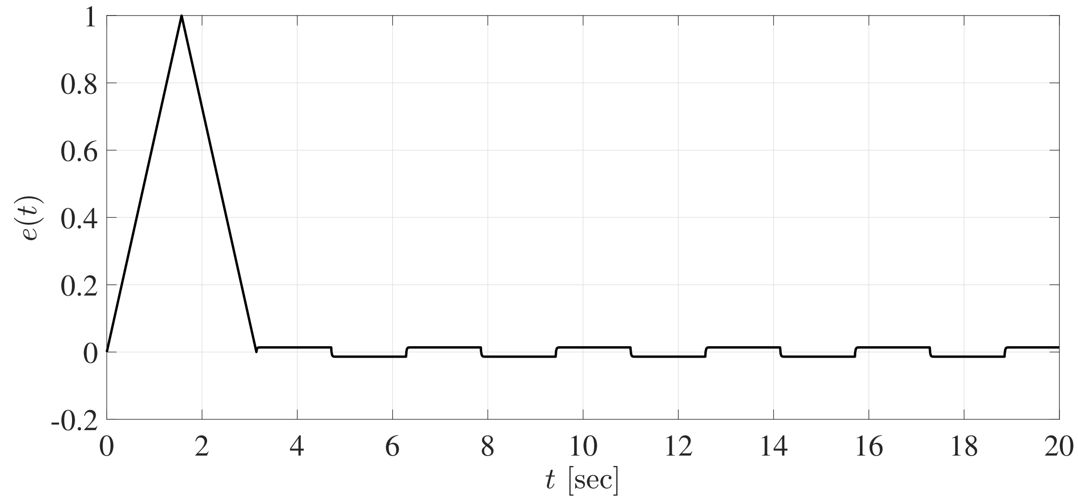

Fig. 3.9: The Response of The Error $e(t)$ in (3.6)

in Figure 3.9. Here, the solid line shows the response of $e(t)$. Figure 3.9 shows that linear functional disturbance observer $\tilde{d}(s)$ in (3.5) for periodic input disturbances could estimate $d(t) - \tilde{d}(t)$ effectively. Although the error is maintained since we cannot completely estimate a disturbance when the plant has an unstable pole.

The study showed that using the parameterization of all linear functional disturbance observers for periodic input disturbances, the study could easily design a linear functional disturbance observer for periodic input disturbances.

3.9 Conclusions

In this chapter, the study proposed parameterizations of all disturbance observers and of all linear functional disturbance observers for periodic input disturbances. The study shows that the proposed method could attenuate periodic disturbances effectively without using repetitive controllers. A design method and a design procedure of linear functional disturbance observers are presented. Finally, the study shows features of the proposed design method through numerical examples.

Chapter 4

Disturbance Observers for Periodic Input and Output Disturbances

4.1 Introduction

In this chapter, the study proposes the parameterization for disturbance observers for periodic input and output disturbances. First, the necessary structure and characteristics of disturbance observers and the linear functional disturbance observer for periodic input and output disturbances are introduced. Next, to attenuate periodic input and output disturbances effectively, a design method for a control system using these parameterizations of all disturbance observers and of all linear functional disturbance observers for periodic input and output disturbance is proposed. In addition, control characteristics of control system using these parameterizations are clarified. A design procedure is also given. Finally, the study offers a numerical example to illustrate the features of the proposed design method. This chapter is organized following: In Section 4.2, the study formulates the problem considered in this thesis. In Section 4.3, the study clarifies the conditions to estimate the periodic output disturbances. In Section 4.4, the study proposes the parameterization of all disturbance observers for periodic output disturbances. In Section 4.5, the study defines the parameterization of all linear functional disturbance observers for periodic output disturbances. In Section 4.6, the study shows a design method for the linear functional disturbance observer. In Section 4.7, the study presents a procedure for linear functional disturbance observers for periodic output disturbances. In Section 4.8, the study provides a numerical example to illustrate the features of the proposed method. In Section 4.9 gives concluding remarks. Therefore, this research could provide a different disturbance observers parameterization and this could attenuate periodic disturbances effectively without using repetitive controllers.

4.2 Problem Formulation

Consider the plant described by

$$\left\{ \begin{array}{rcl} \dot{x}\left(t\right) & = & Ax\left(t\right) + B(u(t) + d_1(t)) \\ y\left(t\right) & = & Cx\left(t\right) + d_2(t) \end{array} \right. , \tag{4.1}$$

where $A \in R^{n\times n}$, $B \in R^{n\times p}$ and $C \in R^{m\times n}$, $x \in R^n$ is the state variable, $u \in R^p$ is the control input, $y \in R^m$ is the output, $d_1(t) \in R^m$ and $d_2(t) \in R^m$ are periodic disturbances with period $T > 0$ satisfying

$$d_1(t+T) = d_1(t) \quad (\forall t \geq 0) \tag{4.2}$$

and

$$d_2(t+T) = d_2(t) \quad (\forall t \geq 0)\,. \tag{4.3}$$

It is assumed that (A, B) is stabilizable and (C, A) is detectable, A has no eigenvalue on the imaginary axis and $u(t)$ and $y(t)$ are available, but $d_1(t)$ and $d_2(t)$ are unavailable. The transfer function from $u(s)$ to $y(s)$ in (4.1) is denoted by

$$y(s) = G(s)u(s) + G(s)d_1(s) + d_2(s), \tag{4.4}$$

where

$$G\left(s\right) = C\left(sI - A\right)^{-1} B \in R^{m\times p}\left(s\right). \tag{4.5}$$

When the disturbances $d_1(t)$ and $d_2(t)$ are unavailable, a disturbance estimator called the disturbance observer is frequently used. The disturbance observer estimates the disturbance of the periodic input disturbance $d_1(t)$

and periodic output disturbance $d_2(t)$ by using an available measurement. Since the available measurements of the plant in (4.1) are $u(t)$ and $y(t)$, that is, $d_1(s)$ and $d_2(s)$ are estimated by the form in

$$\tilde{d}(s) = F_1(s)e^{-sT}y(s) + F_2(s)e^{-sT}u(s), \tag{4.6}$$

where $F_1(s) \in RH_\infty^{m\times m}$, $F_2(s) \in RH_\infty^{m\times p}$, $u(s) = \mathcal{L}\{u(t)\}$, $y(s) = \mathcal{L}\{y(t)\}$, $\tilde{d}(s) = \mathcal{L}\{\tilde{d}(t)\}$ and $\tilde{d}(t) \in R^m$. The structure of disturbance observer $\tilde{d}(s)$ in (4.6) is shown in Figure 4.1.

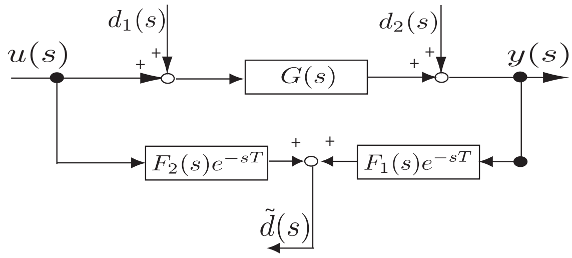

Fig. 4.1: Structure of a disturbance observer

The concept of a disturbance observer for periodic input and output disturbances is proposed following:

Definition *(disturbance observer for periodic input and output disturbances)*
The study calls the system $\tilde{d}(s)$ in (4.6) a "disturbance observer for periodic input and output disturbances", if the error $e(t)$ written by

$$e(t) = \mathcal{L}^{-1}\{G(s)d_1(s)\} + d_2(t) - \tilde{d}(t) \tag{4.7}$$

satisfies

$$\lim_{t\to\infty} e(t) = 0 \tag{4.8}$$

for any initial state $x(0)$, control input $u(t)$ and periodic disturbances $d_1(t)$ and $d_2(t)$.

The problem considered in this thesis is to obtain the parameterization of all disturbance observers $\tilde{d}(s)$ in (4.6) for the periodic input and output disturbances.

4.3 Condition to Estimate The Periodic Input and Output Disturbances

In this section, the study clarifies the condition of $\tilde{d}(s)$ in (4.6) to satisfy (4.8).

The condition of $\tilde{d}(s)$ in (4.6) satisfying (4.8) is summarized in the following theorem.

Theorem 4.3.1 *$\tilde{d}(s)$ in (4.6) works as a disturbance observer for periodic input and output disturbances if and only if*

$$F_1(s)N(s) + F_2(s)D(s) = 0. \tag{4.9}$$

and

$$\left(1 - e^{-s_iT}\right)e(s_i) = 0 \;\; \forall s_i, \tag{4.10}$$

where

$$s_i = j\omega_i \tag{4.11}$$

and

$$\omega_i = \frac{2\pi i}{T} \;\; (i = 0, 1, \ldots) \tag{4.12}$$

and j is the imaginary unit.

Proof: First the necessity is shown, that is, if $\tilde{d}(s)$ in (4.6) satisfies (4.8), then (4.9) and (4.10) are satisfied. $\tilde{d}(s)$ in (4.6) is rewritten by

$$\begin{aligned}\tilde{d}(s) &= (F_1(s)e^{-sT}N(s) + F_2(s)e^{-sT}D(s))\xi(s) + F_1(s)G(s)e^{-sT}d_1(s) \\ &\quad + F_1(s)e^{-sT}d_2(s),\end{aligned} \tag{4.13}$$

where $\xi(s)$ is the pseudo-state variable satisfying

$$u(s) = D(s)\xi(s), \tag{4.14}$$

$N(s) \in RH_\infty{}^{m\times p}$ and $D(s) \in RH_\infty{}^{m\times m}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = N(s)D^{-1}(s). \tag{4.15}$$

$\xi(s)$ in (4.13) is factorized as

$$\begin{aligned}\xi(s) &= \tilde{\xi}(s) + \bar{\xi}(s) \\ &= \frac{1}{1-e^{-sT}}\tilde{\xi}(s) + \bar{\xi}(s),\end{aligned} \tag{4.16}$$

where $\xi(s)$ is denoted by

$$\tilde{\xi}(s) = \int_0^T e^{-sT}\xi(\tau)d\tau, \tag{4.17}$$

$\tilde{\xi}(s)/(1-e^{-sT})$ means the periodic signal with period T and $\bar{\xi}(s)$ includes all other signals. From (4.2) and (4.3), $d_1(s)$ and $d_2(s)$ in (4.13) are written by

$$d_1(s) = \frac{1}{1-e^{-sT}}\hat{d}_1(s) \tag{4.18}$$

and

$$d_2(s) = \frac{1}{1-e^{-sT}}\hat{d}_2(s), \tag{4.19}$$

where

$$\hat{d}_i(s) = \int_0^T e^{-s\tau}d_i(\tau)d\tau \quad (i = 1, 2). \tag{4.20}$$

From (4.13), (4.18) and (4.19), $e(t)$ in (4.7) is given by

$$\begin{aligned}e(s) &= G(s)d_1(s) + d_2(s) - \tilde{d}(s) \\ &= \left(I - F_1(s)e^{-sT}\right)G(s)\frac{1}{1-e^{-sT}}\hat{d}_1(s) + \left(I - F_1(s)e^{-sT}\right)\frac{1}{1-e^{-sT}}\hat{d}_2(s) \\ &\quad - \left(F_1(s)N(s) + F_2(s)D(s)\right)\frac{e^{-sT}}{1-e^{-sT}}\tilde{\xi}(s) \\ &\quad - \left(F_1(s)N(s) + F_2(s)D(s)\right)e^{-sT}\bar{\xi}(s)\end{aligned} \tag{4.21}$$

From the assumption that $e(t)$ satisfies (4.8) for any $\bar{\xi}(s)$,

$$\left(F_1(s)N(s) + F_2(s)D(s)\right)e^{-sT}\bar{\xi}(s) = 0 \tag{4.22}$$

is satisfied for any $\bar{\xi}(s)$. That is, we have (4.9). Substitution of (4.9) to (4.21) gives

$$e(s) = \left(I - F_1(s)e^{-sT}\right)G(s)\frac{1}{1-e^{-sT}}\hat{d}_1(s) + \left(I - F_1(s)e^{-sT}\right)\frac{1}{1-e^{-sT}}\hat{d}_2(s). \tag{4.23}$$

From the assumption that $e(t)$ satisfies (4.8) and internal model principle [41, 43, 44], (4.10) is satisfied. The study has thus proved the necessity.

Next the sufficiency is shown. That is, if (4.9) and (4.10) are satisfied, then $e(t)$ in (4.7) satisfies (4.8). From (4.9), $e(t)$ in (4.7) is written by

$$\begin{aligned}e(s) &= \left(I - F_1(s)e^{-sT}\right)G(s)\frac{1}{1-e^{-sT}}\hat{d}_1(s) + \left(I - F_1(s)e^{-sT}\right)\frac{1}{1-e^{-sT}}\hat{d}_2(s) \\ &\quad - \left(F_1(s)N(s) + F_2(s)D(s)\right)\frac{e^{-sT}}{1-e^{-sT}}\tilde{\xi}(s).\end{aligned} \tag{4.24}$$

From (4.9) and (4.10), $e(t)$ in (4.7) satisfies (4.8). Thus the sufficiency is shown.

The study has thus proved Theorem 4.3.1.

Note that from Theorem 4.3.1, when (4.9) is a condition of disturbance observers for any state variable. In addition, (4.10) is a condition to estimate any periodic signals. Therefore, this is the most important condition to estimate the periodic disturbances and the mentioned condition could solve the problem.

In this section, the study obtained the conditions to estimate the periodic input and output disturbances. In the next section, using the result of Theorem 4.3.1, the study clarifies the parameterization of all disturbance observers for periodic disturbances.

4.4 Parameterization of All Disturbance Observers for Periodic Disturbances

In section, the study proposes the parameterization of all disturbance observer $\tilde{d}(s)$ in (4.6) for periodic input and output disturbance.

The parameterization is summarized in the following theorem.

Theorem 4.4.1 *The system $\tilde{d}(s)$ in (4.6) is the disturbance observer for periodic input and output disturbance if and only if $F_1(s)$ and $F_2(s)$ are written by*

$$F_1(s) = \tilde{D}(s) + Q(s)\tilde{D}(s) \tag{4.25}$$

and

$$F_2(s) = -\tilde{N}(s) - Q(s)\tilde{N}(s) \in RH_\infty^{m\times p}, \tag{4.26}$$

where $\tilde{D}(s) \in RH_\infty^{m\times m}$ and $\tilde{N}(s) \in RH_\infty^{m\times p}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \tilde{D}^{-1}(s)\tilde{N}(s), \tag{4.27}$$

respectively. $Q(s) \in RH_\infty$ is any function satisfying

$$\tilde{D}(s_i) + Q(s_i)\tilde{D}(s_i) = I \ \ \forall s_i (i = 0, \ldots). \tag{4.28}$$

Proof of Theorem 4.4.1 requires the following lemma.

Lemma 4.4.1 *[24] Assume that $A(s) \in RH_\infty^{m\times n}$, $B(s) \in RH_\infty^{q\times p}$, $C(s) \in RH_\infty^{m\times p}$ and*

$$\text{rank} \begin{bmatrix} A^T(s) & B^T(s) \end{bmatrix} = \gamma. \tag{4.29}$$

are satisfied. There exist $X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ satisfying

$$X(s)A(s) + Y(s)B(s) = C(s) \tag{4.30}$$

if and only if there exists $U(s) \in \mathcal{U}$ satisfying

$$\begin{bmatrix} A(s) \\ B(s) \\ C(s) \end{bmatrix} = U(s) \begin{bmatrix} A(s) \\ B(s) \\ 0 \end{bmatrix}. \tag{4.31}$$

When $X_0(s) \in RH_\infty$ and $Y_0(s) \in RH_\infty$ are solution to (4.30), then all solutions to (4.30) are given by

$$\begin{bmatrix} X(s) & Y(s) \end{bmatrix} = \begin{bmatrix} X_0(s) & Y_0(s) \end{bmatrix} + Q(s) \begin{bmatrix} W_1(s) & W_2(s) \end{bmatrix}, \tag{4.32}$$

where $W_1(s)$ and $W_2(s)$ satisfy

$$W_1(s)A(s) + W_2(s)B(s) = 0 \tag{4.33}$$

and

$$\text{rank} \begin{bmatrix} W_1(s) & W_2(s) \end{bmatrix} = n + q - \gamma \tag{4.34}$$

and $Q(s) \in RH_\infty^{p\times(N+q-\gamma)}$ is any function.

Using Theorem 4.3.1 and Lemma 4.4.1, Theorem 4.4.1 is proved.
Proof: From Theorem 4.3.1, $\tilde{d}(s)$ works a disturbance observer for periodic disturbance if and only if $F_1(s) \in RH_\infty^{m\times m}$ and $F_2(s) \in RH_\infty^{m\times p}$ satisfy (4.9). From Lemma 4.4.1, all solution of $F_1(s)$ and $F_2(s)$ to satisfy (4.9) are given by (4.25) and (4.26), respectively, since

$$\tilde{D}(s)N(s) - \tilde{N}(s)D(s) = 0, \tag{4.35}$$

and Lemma 4.4.1, where $\tilde{D}(s) \in RH_\infty^{p\times p}$ and $\tilde{N}(s) \in RH_\infty^{p\times m}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \tilde{D}(s)^{-1}(s)\tilde{N}(s). \tag{4.36}$$

The rest is to prove $\tilde{d}(s)$ in (4.6) works as a periodic input and output disturbance observer if and only if $Q(s)$ in (4.25) and (4.26) satisfy (4.28). From Theorem 4.3.1, $\tilde{d}(s)$ in (4.6) works as a periodic input and output disturbance observer if and only if $e(s)$ in (4.7) satisfy (4.10). The necessity is shown. That is if $\tilde{d}(s)$ in (4.6) works as a periodic input and output disturbance observer, then $Q(s)$ in (4.25) and (4.26) satisfy (4.28). From (4.25) and (4.26), $e(s)$ in (4.7) is written by

$$\begin{aligned} e(s) &= \left\{I - F_1(s)e^{-sT}\right\} G(s) \frac{1}{1-e^{-sT}} \hat{d}_1(s) \\ &\quad + \left\{I - F_1(s)e^{-sT}\right\} \frac{1}{1-e^{-sT}} \hat{d}_2(s). \end{aligned} \tag{4.37}$$

This equation yields

$$\begin{aligned} \left(1-e^{-s_iT}\right) e(s_i) &= \left\{I - \left(\tilde{D}(s_i) + Q(s_i)\tilde{D}(s_i)\right)\right\} G(s)\tilde{d}_1(s_i) \\ &\quad + \left\{I - \left(\tilde{D}(s_i) + Q(s_i)\tilde{D}(s_i)\right)\right\} \tilde{d}_2(s_i) \\ &= 0. \end{aligned} \tag{4.38}$$

The study has (4.28). Thus the study proved the necessity.

Next, the sufficiency is shown. That is, the study shows that if $Q(s)$ in (4.25) and (4.26) satisfy (4.28), then (4.10) is satisfied. $e(s)$ in (4.7) is written by (4.37). Substituting (4.28) to (4.37), it is obvious that (4.10) is satisfied. In this way, the sufficiency has been proved.

From the above discussion, the study has thus proved Theorem 4.4.1. Note that from Theorem 4.4.1, when $G(s)$ is stable, if $Q(s)$ is settled by

$$Q(s) = \tilde{D}^{-1}(s) - I, \tag{4.39}$$

then $Q(s)$ in (4.39) satisfies (4.28). However when $G(s)$ is unstable, it is difficult to set $Q(s)$ satisfying (4.28). For the unstable plant $G(s)$, a disturbance observer for periodic input and output disturbances is often used to attenuate disturbances effectively in [42], even if the system $\tilde{d}(s)$ in (4.6) satisfying (4.10) could not be designed. This means that in order to attenuate periodic disturbances, it is enough to estimate $(I - F(s))G(s)\tilde{d}_1(s) + (I - F(s))\tilde{d}_2(s)$, where $F(s) \in RH_\infty$ is any function. From this point of view, in the next section, when $G(s)$ is unstable, the study defines a linear functional disturbance observer for periodic input and output disturbance observer and clarifies the parameterization of disturbance observer for periodic input and output observers for periodic input and output disturbances.

4.5 Parameterization of All Linear Functional Disturbance Observers for Periodic Input and Output Disturbances

In this section, the study defines a linear functional disturbance observer and presents the parameterization of all linear functional disturbance observers for periodic input and output disturbances.

The study calls $\tilde{d}(s)$ in (4.6) the linear functional disturbance observer for periodic input and output disturbances if $\tilde{d}(s)$ is written by

$$\left(1-e^{-s_iT}\right) e(s_i) = F(s_i)G(s_i)\hat{d}_1(s_i) + F(s_i)\hat{d}_2(s_i) \tag{4.40}$$

is satisfied, where $F(s) \in RH_\infty$ is any function satisfying

$$\bar{\sigma}\left\{F(s_i)\right\} \simeq 0 \ \ \forall s_i (i = 1, \ldots, n_{max}) \tag{4.41}$$

and n_{max} is the maximum frequency satisfying (4.41). Since the available measurements of the plant $G(s)$ in (4.1) are $u(t)$ and $y(t)$ and the input disturbance $d_1(t)$ satisfies (4.2) and the output disturbance $d_2(t)$ satisfies (4.3), the periodic disturbance d(t) is estimated by the form in (4.6), where $F_1(s) \in RH_\infty^{m\times m}$ and $F_2(s) \in RH_\infty^{m\times p}$.

The parameterization of the linear functional disturbance observer for periodic input and output disturbance is summarized following.

Theorem 4.5.1 *The system $\tilde{d}(s)$ in (4.6) is the linear functional disturbance observer for periodic input and output disturbance if and only if $F_1(s)$, $F_2(s)$ and $F(s)$ are described by*

$$F_1(s) = \tilde{D}(s) + Q(s)\tilde{D}(s), \tag{4.42}$$

$$F_2(s) = -\tilde{N}(s) - Q(s)\tilde{N}(s), \tag{4.43}$$

and

$$\begin{aligned} F(s) &= I - F_1(s) \\ &= I - \left(\tilde{D}(s) + Q(s)\tilde{D}(s)\right), \end{aligned} \tag{4.44}$$

respectively, where

$$\bar{\sigma}\left(I - F_1(s_i)\right) = \bar{\sigma}\left\{I - \left(\tilde{D}(s_i) + Q(s_i)\tilde{D}(s_i)\right)\right\} \simeq 0 \ \ \forall s_i (i = 1, \ldots, n_{max}), \tag{4.45}$$

respectively.

Proof: First, the necessity is shown. That is, the study shows that if the system $\tilde{d}(s)$ in (4.6) is a linear functional disturbance observer for a periodic input and output disturbances, then (4.42), (4.43), (4.44) and (4.45) are satisfied. From (4.6), (4.13), (4.14), (4.15), (4.16) and (4.17), for the system $\tilde{d}(s)$ in (4.6), $e(s)$ is written as (4.21). From the assumption that $e(s)$ satisfies (4.8) for any $\bar{\xi}(s)$, (4.23) holds for any $\bar{\xi}(s)$. That is, the study has (4.10). From (4.35) and Lemma 4.4.1, all solutions of $F_1(s)$ and $F_2(s)$ to satisfy (4.10) are given by (4.42) and (4.43), respectively. Substitution of (4.10) to (4.23) gives (4.24). From (4.24) the assumption that $e(s)$ satisfies (4.40), we have (4.44) and (4.45). In this way, the necessity has been proved.

Next, the sufficiency is shown. That is, the study shows that if (4.42), (4.43), (4.44) and (4.45) is satisfied, then the $\tilde{d}(s)$ is a linear functional disturbance observer. Since $e(s)$ in (4.7) is written by (4.21), Substituting (4.42), (4.43), (4.44) and (4.45) to (4.21), it is obvious that (4.40) is satisfied. In this way, the sufficiency has been proved.

From the above, the study has thus proved Theorem 4.5.1.

4.6 Design Method for Linear Functional Disturbance Observers

In this section, the study shows a design method for a linear functional disturbance observer $\tilde{d}(s)$.

In order to design the linear functional disturbance observer $\tilde{d}(s)$ for periodic input and output disturbances, $Q(s)$ in (4.42) and (4.43) needs to satisfy (4.45).

When $G(s)$ is unstable, $Q(s)$ is set as

$$Q(s) = \hat{Q}(s)\left(I - \tilde{D}(s)\right)\tilde{D}_o^{-1}(s), \tag{4.46}$$

where $\tilde{D}_o(s) \in RH_\infty^{m\times m}$ is an outer function of $\tilde{D}(s)$ satisfying

$$\tilde{D}(s) = \tilde{D}_o(s)\tilde{D}_i(s), \tag{4.47}$$

$\tilde{D}_i(s) \in RH_\infty^{m\times m}$ is a co-inner function of $\tilde{D}(s)$ satisfying $\tilde{D}_i(0) = I$ and $\tilde{D}_i(s)\tilde{D}_i(-s)^T = I$, $\hat{Q}(s) \in RH_\infty^{m\times m}$ is any function satisfying

$$\bar{\sigma}\left\{I - \hat{Q}(s_i)\tilde{D}_i(s_i)\right\} \simeq 0 \ \ \forall s_i (i = 1, \ldots, n_{max}). \tag{4.48}$$

4.7 Procedure for Linear Functional Disturbance Observers for Periodic Input and Output Disturbances

In this section, the study shows a design procedure for linear functional disturbance observers for periodic input and output disturbances satisfying Theorem 4.5.1.

A design procedure is summarized following:

<u>Procedure</u>

Step 1) Obtain coprime factors $\tilde{N}(s) \in RH_\infty^{m\times p}$ and $\tilde{D}(s) \in RH_\infty^{m\times m}$ of $G(s) \in R(s)^{m\times p}$ satisfying (4.27). The parameterization of all linear functional disturbance observers is given by (4.6), where $F_1(s)$, $F_2(s)$ and $F(s)$ are written by (4.42), (4.43) and (4.44), respectively.

Step 2) The maximum frequency range n_{max} in (4.45) to estimate the periodic disturbance $d(s)$ is settled.

Step 3) Factorize $\tilde{D}(s)$ as (4.47) satisfying $\tilde{D}_i(0) = I$.

Step 4) Settle $Q(s) \in RH_\infty^{m\times m}$ satisfying (4.45). In order to satisfy (4.45), $Q(s) \in RH_\infty^{m\times m}$ is set according to (4.46). Here $\hat{Q}(s)$ is a low-pass filter satisfying $\hat{Q}(0) = I$, as

$$\hat{Q}(s) = \text{diag}\left\{\frac{k_1}{(1+s\tau_1)^{\alpha_1}}, \ldots, \frac{k_m}{(1+s\tau_m)^{\alpha_m}}\right\}, \tag{4.49}$$

$\alpha_i (i = 1, 2, \ldots, m)$ is an arbitrary positive integer and $k_i (i = 1, 2, \ldots, m)$ satisfying $\tau_i (i = 1, \ldots, m)$

$$\sigma\left\{I - \hat{Q}(s_i)\tilde{D}_i(s_i)\right\} \simeq 0. \ \forall s_i (i = 1, \ldots, m). \tag{4.50}$$

are real numbers.

Step 5) Substituting $Q(s)$ for (4.42), (4.43) and (4.44), $F_1(s)$, $F_2(s)$ and $F(s)$ are obtained. Then the study could design disturbance observer $\tilde{d}(s)$ for periodic input and output disturbances as (4.6).

4.8 Numerical Example

In this section, the study shows numerical examples to illustrate the effectiveness of the proposed parameterizations.

Firstly, the study shows that the proposed design method of the disturbance observer for the stable plant in this thesis could estimate the periodic disturbance more effectively than the other design method of disturbance observers. To compare the effectiveness of the proposed design method in this thesis, the study shows a result that the disturbance observer designed by using a design method of [5] and a proposed method in this thesis estimates the periodic disturbance for a Single-Input/Single-Output stable plant. Next, the study shows that the linear functional disturbance observer for the periodic disturbances designed by using the proposed design method in this thesis could estimate the periodic disturbances for Single-Input/Single-Output unstable plant.

4.8.1 Numerical example 1. A numerical example of disturbance observers for step disturbance by using a design method in [5] for the stable plant

Consider the problem to estimate the periodic disturbance by designing a disturbance observer using a design method in [5] for stable plant $G(s)$ given as

$$G(s) = \frac{25s+1}{s^2+11s+10}. \tag{4.51}$$

The period T of the periodic disturbance $d(t)$ is

$$T = \pi. \tag{4.52}$$

The disturbance observer is denoted as

$$\tilde{d}(s) = Q(s)G(s)^{-1}y(s) + Q(s)u(s), \tag{4.53}$$

where $Q(s)$ in (4.53) is the filter satisfying $\lim_{s\to 0} Q(s) = 1$. $Q(s)$ in (4.53) is settled by

$$Q(s) = \frac{1}{(1+0.1s)^2}. \tag{4.54}$$

When the control input $u(t)$, periodic input disturbance $d_1(t)$ and periodic output disturbance $d_2(t)$ are given by

$$u(t) = 0, \tag{4.55}$$

$$d_1(t) = \sum_{i=1}^{3} \sin(it) \tag{4.56}$$

and

$$d_2(t) = \sum_{i=1}^{3} \sin(it), \tag{4.57}$$

respectively, the response curve of disturbance is estimated by using a design method [5] for the step disturbance. The response curves of disturbance estimations are shown in Figure 4.2. Here, the dotted line shows the periodic disturbances of $Gd_1(t) + d_2(t)$ and the solid line shows the disturbance observer of $\tilde{d}(t)$. Figure 4.2 shows that the disturbance observer $\tilde{d}(s)$ in (4.53) for step disturbance could not estimate $\tilde{d}(t)$ effectively.

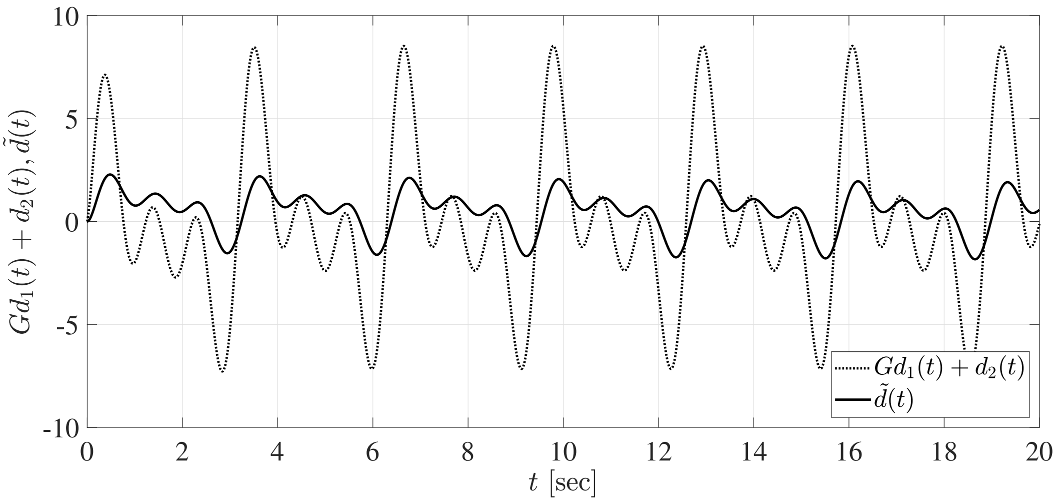

Fig. 4.2: Response curves of the disturbance estimation by using a design method of [5]

4.8.2 Numerical example 2. A numerical example of disturbance observers for step disturbance by using a proposed method for the stable plant

Consider the problem to obtain the parameterization of all disturbance observers for stable plant $G(s)$ written by

$$G(s) = \frac{25s+1}{s^2+11s+10} \tag{4.58}$$

The period T of the periodic disturbance $d(t)$ is

$$T = \pi. \tag{4.59}$$

Coprime factorization of $G(s)$ in (4.58) satisfying (4.27) is given by

$$\tilde{N}(s) = G(s) = \frac{25s+1}{s^2+11s+10} \tag{4.60}$$

and

$$\tilde{D}(s) = 1. \tag{4.61}$$

From Theorem 4.4.1, the parameterization of all disturbance observers $\tilde{d}(s)$ for stable plant $G(s)$ in (4.58) is given by (4.6), where

$$F_1(s) = 1 + Q(s), \tag{4.62}$$

$$F_2(s) = -\frac{25s+1}{s^2+11s+10} - Q(s)\frac{25s+1}{s^2+11s+10} \tag{4.63}$$

and $Q(s) \in RH_\infty$ is any function.

Next using obtained parameterization, we design a disturbance observer $\tilde{d}(s)$ for the periodic input and output disturbances, that is, $Q(s)$ is settled satisfying (4.28). In order to satisfy (4.28), $Q(s)$ is settled by (4.39).

When the control input $u(t)$, periodic input disturbance $d_1(t)$ and periodic output disturbance $d_2(t)$ are given by

$$u(t) = 0, \tag{4.64}$$

$$d_1(t) = \sum_{i=1}^{3} \sin(it) \tag{4.65}$$

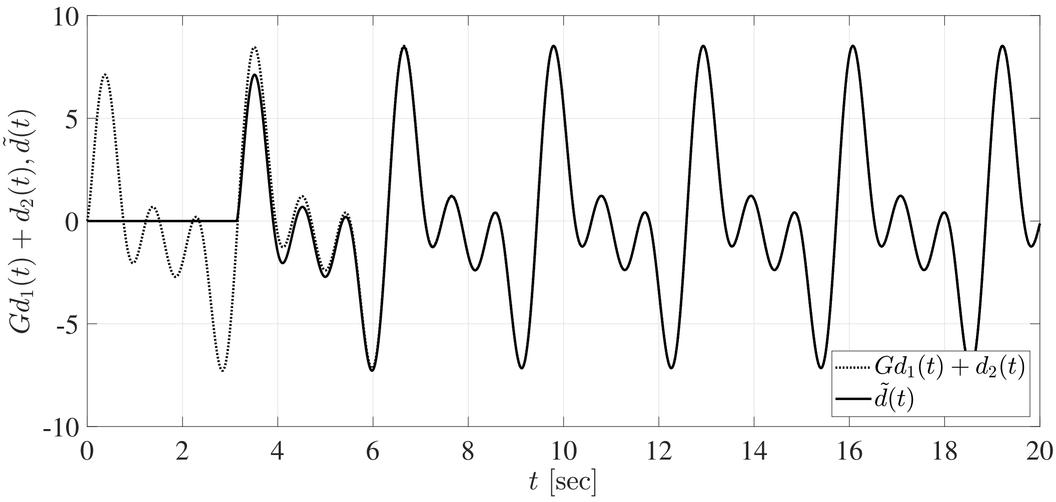

Fig. 4.3: Response curves of the disturbance estimation

and

$$d_2(t) = \sum_{i=1}^{3} \sin(it), \tag{4.66}$$

respectively, the response curves of disturbance is estimated by using a proposed method for the step disturbance. The response curves of disturbance estimations are shown in Figure 4.3. Here, the dotted line shows the periodic disturbances of $Gd_1(t) + d_2(t)$ and the solid line shows the disturbance observer of $\tilde{d}(t)$. Figure 4.3 shows that disturbance observer $\tilde{d}(s)$ in (4.6) for step disturbance could estimate $\tilde{d}(t)$ effectively.

In this way, it is shown that using the obtained parameterization of all disturbance observers for periodic input and output disturbances, the study could easily design a disturbance observer for step disturbance.

4.8.3 Numerical example 3. A numerical example for disturbance observers

Consider the problem to obtain the parameterization of all disturbance observers for stable plant $G(s)$ written by

$$G(s) = \frac{25s+1}{s^2+11s+10} \tag{4.67}$$

The period T of the periodic disturbance $d(t)$ is

$$T = \pi. \tag{4.68}$$

Coprime factorization of $G(s)$ in (4.67) satisfying (4.27) is given by

$$\tilde{N}(s) = G(s) = \frac{25s+1}{s^2+11s+10} \tag{4.69}$$

and

$$\tilde{D}(s) = 1. \tag{4.70}$$

From Theorem 4.4.1, the parameterization of all disturbance observers $\tilde{d}(s)$ for stable plant $G(s)$ in (4.67) is given by (4.6), where

$$F_1(s) = 1 + Q(s), \tag{4.71}$$

$$F_2(s) = -\frac{25s+1}{s^2+11s+10} - Q(s)\frac{25s+1}{s^2+11s+10} \tag{4.72}$$

and $Q(s) \in RH_\infty$ is any function.

Next using obtained parameterization, the study design a disturbance observer $\tilde{d}(s)$ for the periodic input and output disturbances, that is, $Q(s)$ is settled satisfying (4.28). In order to satisfy (4.28), $Q(s)$ is settled by (4.39).

When the control input $u(t)$, periodic input disturbance $d_1(t)$ and periodic output disturbance $d_2(t)$ are given by

$$u(t) = 0, \tag{4.73}$$

$$d_1(t) = \sum_{i=1}^{3} \sin(it) \tag{4.74}$$

and

$$d_2(t) = \sum_{i=1}^{3} \sin(it), \tag{4.75}$$

respectively, the response of the error $e(t)$ in (4.7) is shown in Fig. 4.4. Here, the solid line shows the response

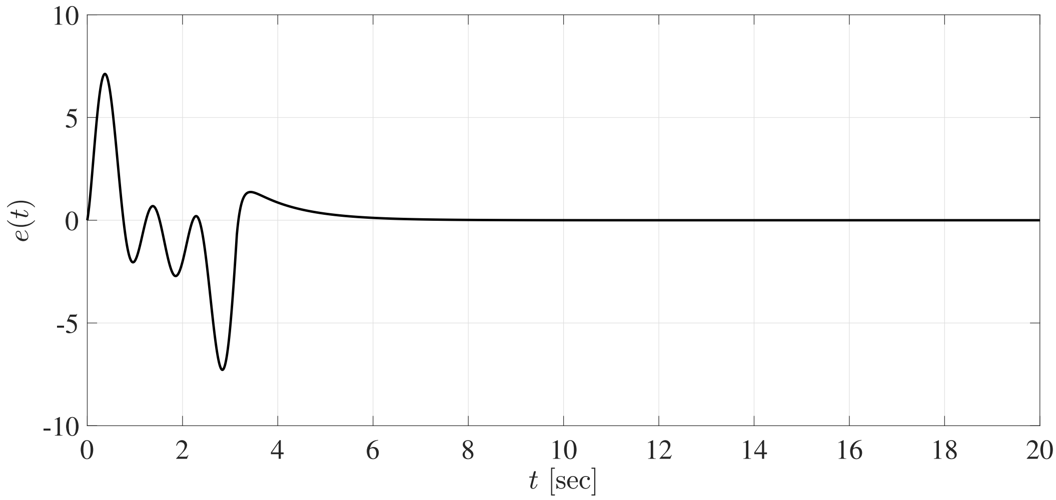

Fig. 4.4: The response of the error $e(t)$ in (4.7)

of $e(t)$. Fig. 4.4 shows that disturbance observer $\tilde{d}(s)$ in (4.6) for periodic input and output disturbances could estimate $\mathcal{L}^{-1}\{G(s)d_1(s)\} + d_2(t) - \tilde{d}(t)$ effectively.

In this way, it is shown that using the obtained parameterization of all disturbance observers for periodic input and output disturbances, the study could easily design the disturbance observer for periodic input and output disturbances.

4.8.4 Numerical example 4. A numerical example for linear functional disturbance observers

Consider the problem to obtain the parameterization of all linear functional disturbance observers for periodic input and output disturbances for unstable plant $G(s)$ described by

$$G(s) = \frac{s+1}{s^2 - 94s - 600} \tag{4.76}$$

The period T of the periodic disturbances is

$$T = \pi. \tag{4.77}$$

A pair of coprime factors $\tilde{N}(s) \in RH_\infty$ and $\tilde{D}(s) \in RH_\infty$ of $G(s)$ in (4.76) satisfying (4.27) is given by

$$\tilde{N}(s) = \frac{-2s - 2}{s^2 + 1006s + 6000} \tag{4.78}$$

and

$$\tilde{D}(s) = \frac{-2s+200}{s+1000}, \tag{4.79}$$

respectively. From Theorem 4.5.1, the parameterization of all linear functional disturbance observers $\tilde{d}(s)$ is given by (4.6), where

$$F_1(s) = \frac{-2s+200}{s+1000} + Q(s)\frac{-2s+200}{s+1000}, \tag{4.80}$$

$$F_2(s) = \frac{2s+2}{s^2+1006s+6000} + Q(s)\frac{2s+2}{s^2+1006s+6000}, \tag{4.81}$$

$$F(s) = 1 - \frac{-2s+200}{s+1000} - Q(s)\frac{-2s+200}{s+1000} \tag{4.82}$$

and $Q(s) \in RH_\infty$ is any function.

Next using obtained parameterization, the study design a linear functional disturbance observer $\tilde{d}(s)$ for the periodic input and output disturbances by using the procedure described in Section 4.7, that is, $Q(s)$ is settled satisfying (4.28). The maximum frequency range n_{max} in (4.45) to estimate the periodic disturbance $d(s)$, is settled by

$$n_{max} = 3. \tag{4.83}$$

$\tilde{D}(s)$ in (4.79) is factorized as (4.47), where

$$\tilde{D}_o(s) = \frac{2s+200}{s+1000}, \tag{4.84}$$

and

$$\tilde{D}_i(s) = \frac{-s+100}{s+100}. \tag{4.85}$$

In order to satisfy (4.45), $\hat{Q}(s)$ is settled by

$$\hat{Q}(s) = 1. \tag{4.86}$$

In order to confirm that $\hat{Q}(s)$ in (4.86) satisfy (4.45), the study shows the gain plot of $1 - \hat{Q}(s)\tilde{D}_i(s)$ in Fig. 4.5. Figure 4.5 shows $\hat{Q}(s)$ in (4.86) satisfies (4.45). $Q(s)$ is set by (4.46) and written by

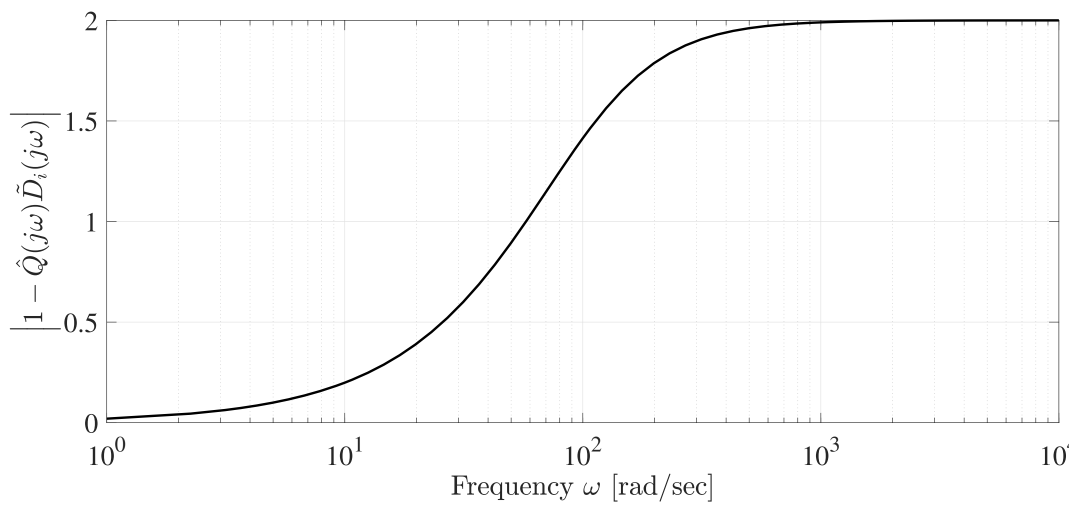

Fig. 4.5: The gain plot of $1 - \hat{Q}(s)\tilde{D}_i(s)$

$$Q(s) = \frac{1.5s+400}{s+100}. \tag{4.87}$$

From (4.42), (4.43) and (4.44), we have $F_1(s)$, $F_2(s)$ and $F(s)$ are designed as

$$F_1(s) = \frac{-5s+500}{s+1000}, \tag{4.88}$$

$$F_2(s) = \frac{5s^2+1005s+1000}{s^3+1106s^2+106600s+600000} \tag{4.89}$$

and

$$F(s) = \frac{6s+500}{s+1000}. \tag{4.90}$$

When the control input $u(t)$, periodic input disturbance $d_1(t)$ and periodic output disturbance $d_2(t)$ are given by

$$u(t) = 0, \tag{4.91}$$

$$d_1(t) = \sum_{i=1}^{3} \sin(it) \tag{4.92}$$

and

$$d_2(t) = \sum_{i=1}^{3} \sin(it) \tag{4.93}$$

the response of the error $e(t)$ in (4.7) is shown in Fig. 4.6. Here, the solid line shows the response of $e(t)$. Fig.

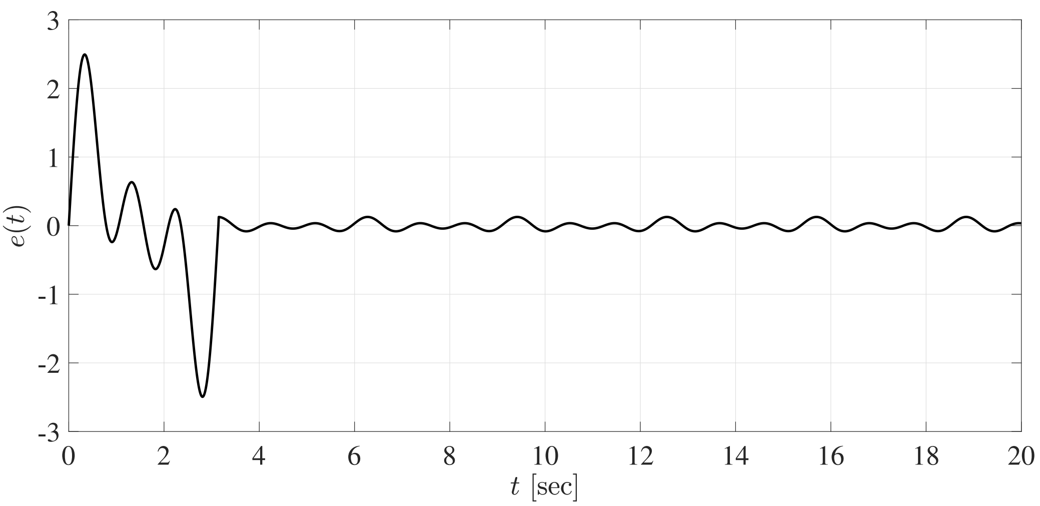

Fig. 4.6: The response of the error $e(t)$ in (4.7)

4.6 shows that linear functional disturbance observer $\tilde{d}(s)$ in (4.6) for periodic input and output disturbances could estimate $\mathcal{L}^{-1}\{G(s)d_1(s)\} + d_2(t) - \tilde{d}(t)$ effectively.

The study has thus shown that using the parameterization of all linear functional disturbance observers for periodic input and output disturbances, the study could easily design a linear functional disturbance observer for periodic input and output disturbances.

4.9 Conclusions

In this chapter, the study has proposed parameterizations of all disturbance observers and of all linear functional disturbance observers for periodic input and output disturbances. The study shows that the proposed method could attenuate periodic disturbances effectively without using repetitive controllers. A design method and a design procedure of linear functional disturbance observer are presented. Finally, the study shows features of the proposed design method that were illustrated through numerical examples.

Chapter 5

Conclusions

This chapter presented the conclusion of the study. The results of this thesis could be summarized following:

In chapter 2, the study explained parameterizations of all disturbance observers and of all linear functional disturbance observers for periodic output disturbances. Next, the study shows that the proposed method could attenuate periodic disturbances effectively without using repetitive controllers. In addition, a design method and a design procedure of linear functional disturbance observers are presented. Finally, the study shows features of the proposed design method through numerical examples.

In chapter 3, the study explained parameterizations of all disturbance observers and of all linear functional disturbance observers for periodic input disturbances. Next, the study shows that the proposed method could attenuate periodic disturbances effectively without using repetitive controllers. In addition, a design method and a design procedure of linear functional disturbance observers are presented. Finally, the study shows features of the proposed design method through numerical examples.

In chapter 4, the study described parameterizations of all disturbance observers and of all linear functional disturbance observers for periodic input and output disturbances. Next, the study shows that the proposed method could attenuate periodic disturbances effectively without using repetitive controllers. In addition, a design method and a design procedure of linear functional disturbance observer are presented. Finally, the study shows features of the proposed design method that was illustrated through numerical examples.

In future research, Using obtained parameterizations, a design method of control systems would be discussed in another research.

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