令和５年度　博士論文

Study on minimum-phase controllers for minimum-phase plants

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Chapter 1

Introduction

1.1 A trend of a study for parameterization problem

There exist a important control problem named the parameterization problem, which is to seek all stabilizing controllers for a plant [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] and to obtain plants those can be stabilized [11, 12, 13, 14].

In the parameterization problem, the search for the controlled plants that can be stabilized is the first step to be considered. Usually we find a defined parameterization of the controlled plants based on the form of the system. Based on the parameterization of the plants that can be stabilized, we further determine the parameterization of the stabilizing controllers.

Using the parameterization, the stability of the control system is first ensured by selecting a controller from the parameterization. The parameterization can be freely chosen as a free parameter. We can meet specifications other than stability by using the flexibility of this free parameter. In other words, we can propose a controller design method based on the parameterization of the stabilizing controllers to satisfiy stability and other specifications. Thus, we can easily design the desired stabilizing controller. Obtaining the parameterization is therefore an important control problem.

1.1.1 YK parameterization

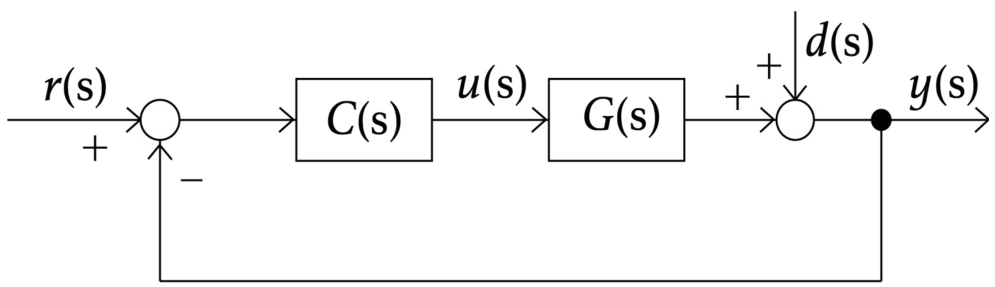

Fig. I: Feedback control system

The parametrization problem have been tackled in many works using the Youla Kucera parametrization [1]. Youla proposed a parameterization that provides all linear stabilizing controllers for a given Linear Time-Invariant (LTI) plant in a feedback control loop in in Figure I. All stabilizing controllers are parametrized based on the transfer function called YK parameter Q, leading to a control form $C(Q)$. The parameter $Q(s)$ is the one guaranteeing the stability. Similarly, its dual theory (also known as the dual YK parametrization) provides all the linear plants stabilized by a given controller. The class of all the plant stabilized by a controller depends on the transfer function called dual YK parameter S,so $G(S)$. This parameter could represent any plant variations. Hence, this useful way of parametrizing either plants, controllers or both is employed to solve many control issues. According to the control objectives, two main configurations can be targeted:

- Controller parametrization allows stable controller reconfiguration when some change occurs. It is also widely used in disturbance and noise rejection control.
- Plant parametrization is employed to solve the problem of closed-loop identification.

Consider a single-input single-output (SISO) stable plant $G(s)$ connected to a given controller $C(s)$ in a stable feedback loop depicted as in Figure I. Closed-loop transfer function $T(s)$ from reference $r(s)$ to output $y(s)$ is in the following equation:

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)} \tag{1.1}$$

The transfer function from the reference $r(s)$ to the controller input $u(s)$ yields:

$$Q(s) = \frac{C(s)}{1 + C(s)G(s)} \tag{1.2}$$

if Q(s) and G(s) are known, the controller $C(s)$ can be expressed as:

$$C(s) = \frac{Q(s)}{1 - G(s)Q(s)} \tag{1.3}$$

If $C(s)$ is stabilizing $G(s)$, $Q(s)$ is stable and proper. Reciprocally, if $Q(s)$ is stable and proper, it is easy to demonstrate that $C(s)$ stabilizes $G(s)$ using (1.3). Thus, stabilizing controllers can be parametrized in terms of the set of all stable proper functions $Q(s)$ for a given plant $G(s)$.

Conversely, a dual concept is proposed by leading the same reasoning in (1.2) and (1.3) and use the fact that $C(s)$ and $G(s)$ are commutative. The set of all plants stabilized by a given stabilizing controller is characterized using the so called dual YK parameter $S(s)$ [14].

1.1.2 Strong stablization

In a control system, the zeros and poles of the controller and the plant should be considered. In the feedback control system, a stable controller stabilize a plant to make the control system stable. We call this stabilization as strong stabilization. In fact, not all the plant have strongly stabilizing controllers, that is, not all the plant could be stabilized by a stable controller. So, we should search for the plant which could be strongly stabilized. Youla et al. proposed parity interlacing property condition in 1976 and We can identify the strong stabilizable plant by using this condition. But they just gave a method to identify the strong stabilizable plants, but not the special form of strongly stabilizable plants. From this point of view Hoshikawa et al. clarify the characteristic of strongly stabilizable single-input/single-output plants [13]. And Akuzawa et al. propose the parameterization of all strongly stabilizing controllers for strongly stabilizable single-input/single-output plants[8]. The strong stabilization can be applied to control systems working in an environment where the feedback interconnection is easy to break. Examples are power plants and communication networks [8].

1.2 A trend of a study for minimum-phase system

Compared with the non-minimum phase system, the minimum-phase system has fast response, small energy delay [15], stable inverse system and other advantages [16]. Comparing with the nonminimum-phase system constructed by configuring the right half-plane zero point or adding the time-delay, it is obtained that the minimum-phase system has the shortest response time [16]. At any time, the cumulative output energy of the minimum phase system is not less than that of the non-minimum phase system. It can be proved that the cumulative output energy of the minimum phase system is closer to time 0 and has the shortest energy delay [15]. And the inverse system of the minimum-phase system is stable, because the stable poles of the inverse system of the minimum phase system is the zeros of the original system which has no unstable zero. Benefiting from these advantages, the minimum-phase system is widely used in signal processing [15] and other related fields, such as state system, design of causal stable digital filter [17], neural network and calculation [18] and processing of cepstrum and inverse filtering [19].

1.2.1 Simple parameterization of stabilizing controllers for linear minimum-phase plants

Glaria and Goodwin provided a simple parameterization for the clase of stabilizing controllers for linear single-input single-output minimum-phase plants [4].

The result in [4] is the dual of the well-known parameterization for the class of all stabilizing controllers for linear single-input singleoutput stable plants in (1.3).

According to [4],a single-input single-output plant having transfer function $G(s)$ such that $G^{-1}(s)$ is analytic in the closed right half plane. The closed-loop system shown in Figure I is internally stable if and only if $C(s)$ is parameterized as

$$C(s) = \frac{1}{Q(s)} - \frac{1}{G(s)}, \tag{1.4}$$

where $1/Q(s)$ is any stable nonzero transfer function.

For the parameterization given in (1.4),the sensitivity function $S(s)$and complementary sensitivity function $T(s)$ are provided in $Q(s)$ separately

$$S(s) = \frac{Q(s)}{G(s)}, \tag{1.5}$$

$$T(s) = 1 - \frac{Q(s)}{G(s)}. \tag{1.6}$$

And they added a simple linear constraint to the design to automatically guarantee the properness.

1.2.2 Parameterization of all proper internally stabilizing controllers for minimum-phase systems

Yamada presents a parameterization for the class of all proper stabilizing controllers for linear minimum phase systems [5]. He notes taht the parameterization of the stabilizing controller $C(s)$ given by Glaria and Goodwin generally includes improper controllers and the controller $C(s)$ is required to be proper.

According to [5], if $G(s)$ is of minimum-phase and biporper, then the proper internally stabilizing controller $C(s)$ for the unity feedback control system in Figure I is parametrized as

$$C(s) = \frac{1}{Q(s)} - \frac{1}{G(s)}, \tag{1.7}$$

where $1/Q(s)$ is any minimum-phase and biproper rational function.

On the other hand, if $G(s)$ is assumed to be minimum-phase and strictly proper, there exists $K(s)$ satisfies the following expressions:

- $G(s) + K(s)$ is of minimum-phase.
- $K(s)$ is biproper and asymptotically stable.

The parameterization of all proper stabilizing controllers $C(s)$ for $G(s)$ is given by

$$C(s) = \frac{\bar{C}(s)}{1 + \bar{C}(s)K(s)}, \tag{1.8}$$

where $\bar{C}(s)$ is denoted by

$$\bar{C}(s) = \frac{1}{Q(s)} - \frac{1}{G(s) + K(s)}, \tag{1.9}$$

and the free parameter $1/Q(s)$ is any nonsingular proper minimum-phase rational transfer function.

1.3 The purpose and contents of this study

In this thesis, a study on the parameterization of all minimum-phase stabilizing controllers for minimum-phase plants is considered.

In chapter 2, we propose the parameterization of all minimum-phase stabilizing controllers for minimum-phase stabilizing biproper plants. That is, we consider the parameterization that the stabilizing controller makes minimum-phase biproper plant stable, which the stabilizing controller is of minimum-phase. Analysis of the internal stability of closed-loop system are considered. We also present a design method of the minimum-phase stabilizing controllers for the minimum-phase biproper plants that satisifies the robustness of the control system according to the internal model principle. In addition, we show a numerical example to illustrate the features of the proposed design method. In order to check the robustness of this example, we considered this situation that the minimum-phase controller stabilizes the perturbed plant. And we compare the proposed design method with that of [25].

In chapter 3, we propose the parameterization of all minimum-phase stabilizing controllers for minimum-phase stabilizing strictly proper plants. That is, we consider the parameterization that the stabilizing controller makes minimum-phase strictly proper plant stable, which the stabilizing controller is of minimum-phase. Analysis of the internal stability and control characteristics of closed-loop system are provided. We also present a design method of the minimum-phase stabilizing controllers that contributes to the construction of a minimum-phase closed-loop system and check the robustness of the proposed design method. In addition, we show a numerical example to illustrate the features of the proposed design method.

In chapter 4, we propose the parameterization of all minimum-phase stabilizing controllers for minimum-phase stabilizing multiple-input multiple-output plants. In this chapter, we extend the conclusions of the previous two chapters to multiple-input multiple-output systems. The control system characteristics with the parameterization of all stabilizing minimum-phase controllers for minimum-phase biproper multiple-input multiple-output plants is shown. In addition, we propose a design method of stabilizing minimum-phase controllers based on the parameterization of all stabilizing minimum-phase controllers for minimum-phase plants satisifying the robustness. We also show a numerical example to illustrate the features of the proposed design method. At last, we clarify the parameterization of all stabilizing minimum-phase controllers for minimum-phase strictly proper multiple-input multiple-output plants. In this way we obtain the parameterization of all stabilizing minimum-phase controllers for minimum-phase multiple-input multiple-output plants.

In Chapter 5 Summaries the result of the present study by the conclusion.

Chapter 2

Minimum-phase Controllers for Minimum-phase SISO Biproper Plants

2.1 Introduction

Compared with the non-minimum phase system, the minimum-phase system has fast response, small energy delay [15], stable inverse system and other advantages [16]. Comparing with the nonminimum-phase system constructed by configuring the right half-plane zero point or adding the time-delay, it is obtained that the minimum-phase system has the shortest response time [16]. At any time, the cumulative output energy of the minimum phase system is not less than that of the non-minimum phase system. It can be proved that the cumulative output energy of the minimum phase system is closer to time 0 and has the shortest energy delay [15]. And the inverse system of the minimum-phase system is stable, because the stable poles of the inverse system of the minimum phase system is the zeros of the original system which has no unstable zero. Benefiting from these advantages, the minimum-phase system is widely used in signal processing [15] and other related fields, such as state system, design of causal stable digital filter [17], neural network and calculation [18] and processing of cepstrum and inverse filtering [19].

For minimum-phase system, Glaria and Goodwin [4] gave a simple parameterization of controllers for linear minimum-phase plants. However, two difficulties remain. One is that the parameterization of all stabilizing controller given by Glaria and Goodwin generally includes improper controllers. In practical applications, the controller is required to be proper. The other is that they do not give the parameterization of all internally stabilizing controllers. Yamada overcame these problems and proposed the parameterization of all proper internally stabilizing controllers for linear minimum-phase plants [5]. The parameterization of all stabilizing controllers in [5] is applied to many control problems such as the parameterization of all stabilizing modified repetitive controllers for minimum-phase plants [20], adaptive control systems [21, 22], model feedback control systems [23], parallel compensation technique [24], PI control [25] and PID control [26]. Chen et al. proposed the parameterization of all proper stabilizing internal model controllers for minimum-phase unstable plants. Expanded from the result, the parameterization of all strongly stabilizable plants is clarified in [11, 13].

However, there exists a question whether or not, stabilizing controllers for minimum-phase plants can be of minimum-phase that has advantages such as the inverse system of the minimum-phase system is still stable. The inverse system of the minimum-phase system is stable, because the stable poles of the inverse system of the minimum phase system is the stable zeros of the original system. If we use a minimum-phase controller to stabilize a minimum-phase plant, it would make the sensitivity function of this system lower. And Lower values of sensitivity function suggest further attenuation of the external disturbance. The minimum-phase system is widely used in signal processing and other related fields, such as state system, design of causal stable digital filter, and calculation and processing of cepstrum and inverse filtering. If the parameterization of all stabilizing minimum-phase controllers for minimum-phase plants is clarified, we will obtain a new control method for minimum-phase system. From this viewpoint, it is desirable to clarify the parameterization of all stabilizing minimum-phase controllers for minimum-phase plants.

In this chapter, we propose the parameterization of all stabilizing minimum-phase controllers for minimum-phase biproper plants, that is, we consider the parameterization that the stabilizing controller makes minimum-phase plant stable, which the stabilizing controller is of minimum-phase. This chapter is organized as follows: In Section 2.2, we show the problem considered in this chapter. In Section 2.3, we clarify the parameterization of all stabilizing minimum-phase controllers for minimum-phase biproper plants. And analysis of the internal stability of closed-loop system are provided. In Section 2.6, we show a numerical example to illustrate the features of the proposed design method. In order to check the robustness of this example, we considered this situation that the minimum-phase controller stabilizes the perturbed plant. And we compare the proposed design method with that of [25]. By using the same $Q(s)$, the controller that we obtained by the proposed method is of minimum- phase. In contrast, by using the previous method, the controllers that we obtain are not necessarily of minimum-phase. In Section 2.7 we give concluding remarks.

2.2 Problem Formulation

Consider the control system in

$$\begin{cases} y(s) &= G(s)u(s) \\ u(s) &= C(s)(r(s)-y(s)) \end{cases}, \tag{2.1}$$

where $G(s) \in R(s)$ is the plant, $C(s) \in R(s)$ is the controller, $y(s) \in R(s)$ is the output , $u(s) \in R(s)$ is the control input and $r(s) \in R(s)$ is the reference input. $R(s)$ denotes the set of real rational functions with s. $G(s)$ and $C(s)$ are assumed to be of minimum-phase, that is, $G(s)$ and $C(s)$ have no zero in the closed right half plane. And $G(s)$ is assumed to be biproper. The minimum-phase controller controls the minimum-phase plant to make the closed-loop system in (2.1) stable.

According to [5], if $G(s)$ is of minimum-phase and biproper, then the proper controller $C(s)$ stabilize the feedback control system in (2.1) if and only if $C(s)$ is parametrized as

$$C(s) = \frac{1}{Q(s)} - \frac{1}{G(s)}, \tag{2.2}$$

where $1/Q(s)$ is any minimum-phase and biproper rational function. And $C(s)$ is not necessarily of minimum-phase

The problem considered in this paper is to clarify the parameterization of all stabilizing minimum-phase controllers for minimum-phase plants.

2.3 The Parameterization of All Stabilizing Minimum-Phase Controllers for Minimum-Phase Biproper Plants

In this section, we clarify the parameterization of all stabilizing minimum-phase controllers $C(s)$ for minimum-phase biproper plants $G(s)$.

This parameterization is summarized in the following theorem.

Theorem 2.3.1 *$G(s)$ is assumed to be of minimum-phase and to be biproper. Then the minimum-phase controller $C(s)$ stabilizes the feedback control system in (2.1) if and only if $C(s)$ is written by the form of*

$$C(s) = \frac{Q(s)}{(1-Q(s))G(s)}, \tag{2.3}$$

where $Q(s) \in RH_\infty$ is any minimum-phase function to make $(1-Q(s))G(s) \in RH_\infty$.

(Proof)
First, the necessity is shown. That is, we show that if the minimum-phase controller $C(s)$ makes minimum-phase plant $G(s)$ stable, then $C(s)$ takes the form of (**??**). From the assumption that $C(s)$ in (**??**) makes $G(s)$ in (2.1) stable, $1/(1+C(s)G(s))$, $C(s)/(1+C(s)G(s))$, $G(s)/(1+C(s)G(s))$ and $C(s)G(s)/(1+C(s)G(s))$ are all included in RH_∞. RH_∞ denotes the set of stable proper real rational functions.

From the assumption that $G(s)$ and $C(s)$ are both assumed to be of minimum-phase,

$$\frac{1+C(s)G(s)}{C(s)G(s)} = 1 + \frac{1}{C(s)G(s)} \tag{2.4}$$

is stable. Using $Q(s) \in RH_\infty$, $C(s)G(s)/(1+C(s)G(s)) \in RH_\infty$ can be rewritten as

$$\frac{C(s)G(s)}{1+C(s)G(s)} = Q(s). \tag{2.5}$$

From (2.4) and (2.5), $Q(s)$ must be of minimum-phase. From simple manipulation, (2.5) is rewritten as

$$C(s) = \frac{Q(s)}{(1-Q(s))G(s)}. \tag{2.6}$$

Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, if $C(s)$ takes the form of (**??**), then the minimum-phase controller $C(s)$ stabilizes the minimum-phase plant $G(s)$ to make the control system stable. Then we set $C(s)$ as

$$C(s) = \frac{Q(s)}{(1-Q(s))G(s)}, \tag{2.7}$$

where $Q(s) \in RH_\infty$ is any minimum-phase function and $(1-Q(s))G(s) \in RH_\infty$ is any function. If the controller $C(s)$ makes $G(s)$ stable, according to definition of internal stability, the transfer functions $1/(1 + C(s)G(s))$, $C(s)/(1+C(s)G(s))$, $G(s)/(1+C(s)G(s))$ and $C(s)G(s)/(1+C(s)G(s))$ are stable. After simple manipulation, the transfer functions are rewritten as

$$\frac{1}{1+C(s)G(s)} = 1 - Q(s), \tag{2.8}$$

$$\frac{C(s)}{1+C(s)G(s)} = \frac{Q(s)}{G(s)}, \tag{2.9}$$

$$\frac{G(s)}{1+C(s)G(s)} = (1-Q(s))G(s) \tag{2.10}$$

and

$$\frac{C(s)G(s)}{1+C(s)G(s)} = Q(s). \tag{2.11}$$

Because $Q(s) \in RH_\infty$ and $G(s)$ is of minimum-phase, transfer functions in (2.8), (2.9) and (2.11) are stable. If the transfer function in (2.10) is unstable, unstable poles of the transfer function in (2.10) are unstable poles of $G(s)$. From the assumption that $(1-Q(s))G(s) \in RH_\infty$, unstable poles of $G(s)$ are not poles of $(1-Q(s))G(s)$. Therefore, the transfer function in (2.10) is stable. Thus, the sufficiency has been shown.

We have thus proved Theorem 3.3.1.

2.4 Characteristics of the control system

We elucidate the characteristics of the closed-loop control system using the parameterization of all the stabilizing minimum-phase controllers given by (2.3) in this section.

First, the reference tracking characteristic is considered. Here, using the parameterization of all minimum-phase stabilizing controllers for the minimum-phase plants in (**??**), the transfer function in (3.1) from the reference input $r(s)$ to the output $y(s)$ of the control system is given as

$$\frac{y(s)}{r(s)} = \frac{C(s)G(s)}{1+C(s)G(s)} = Q(s). \tag{2.12}$$

Therefore, in order to make the output $y(s)$ follow the step reference input $r(s) = 1/s$ without steady-state error,

$$Q(0) = 1 \tag{2.13}$$

must be achieved. Thus, the output $y(s)$ follows the step reference input $r(s) = 1/s$ without steady-state error, if $Q(s)$ satisfies the following condition

$$Q(0) = 1. \tag{2.14}$$

Next, we consider the disturbance attenuation characteristic. The transfer function from the disturbance $d(s)$ to the output $y(s)$ is given as

$$\frac{y(s)}{d(s)} = \frac{1}{1+C(s)G(s)} = 1 - Q(s). \tag{2.15}$$

Therefore, in order to fully decay the step disturbance $d(s) = 1/s$,

$$1 - Q(0) = 0 \tag{2.16}$$

must be achieved. Thus, the step disturbance of $d(s) = 1/s$ will effectively be rejected if $Q(s)$ satisfies the following condition

$$Q(0) = 1. \tag{2.17}$$

2.5 Design method of minimum-phase stabilizing controllers

In this section, we propose a design method of stabilizing minimum-phase controllers. First, the concept of the design is considered. In this chapter, the plant is of minimum-phase and biproper. By using the parameterization of stabilizing minimum-phase controllers $C(s)$ for minimum-phase biproper plants $G(s)$, $Q(s)$ should be proper and make $(1 - Q(s))G(s)$ be proper to make the controller be proper. We also need to settle $Q(s)$ should be minimum-phase and make $(1-Q(s))G(s)$ be stable to make the controller be minimum-phase. Furthermore, in order to make the output $y(s)$ follow the reference input $r(s) = 1/s$ without steady-state error and attenuate the step disturbance $d(s) = 1/s$ completely, $Q(s)$ needs to satisfy (2.14) and (2.17). Then a design method for the minimum-phase stabilizing controller $C(s)$ is concluded as follows.

1. Let $G(s)$ be factorized as

$$G(s) = G_1(s)G_2(s), \tag{2.18}$$

   where $G_1(s)$ is biproper and of minimum-phase and contains all unstable poles of $G(s)$ and $G_2(s)$ is biproper and of minimum-phase and contains all stable poles of $G(s)$.

2. Using $G_1(s)$, design $Q(s) \in RH_\infty$ as

$$Q(s) = 1 - \frac{ks}{G_1(s)\,(\tau s + 1)^\alpha}, \tag{2.19}$$

   where $\tau \in R$, α is an arbitrary positive integer to make $Q(s)$ proper, k is a constant and s confirm $Q(0) = 1$

3. Using $Q(s)$ in (2.19), fix a minimum-phase stabilizing controller $C(s)$ in (2.3).

First, we show the parameter $Q(s) \in RH_\infty$ is of minimum-phase. Because $G_1(s)$ in (2.18) is biproper and α is an arbitrary positive integer, $Q(s)$ is proper in (2.19). Since $G_1(s) \in RH_\infty$ contains the unstable poles of $G(s)$, that is G_1 is minimum-phase and unstable, $Q(s)$ is stable and of minimum-phase. Next, we check that $(1 - Q(s))G(s)$ is stable and proper. By substituting (2.19) into $(1 - Q(s))G(s)$,

$$(1 - Q(s))G(s) = G_2(s)\frac{ks}{(\tau s + 1)^\alpha} \tag{2.20}$$

is obtained. As $G_2(s) \in \mathcal{U}$ in (2.18), $(1 - Q(s))G(s)$ is obviously stable and proper. Finally, we find that $Q(s) \in RH_\infty$ defined by (2.19) is guaranteed to satisify (2.14).

2.6 Numerical example

In this section, a numerical example is illustrated to show that a stabilizing minimum-phase controller written by the form of (2.3) can stabilize the minimum-phase plant.

Consider the problem to make the control system in (2.1) stable using stabilizing minimum-phase controller, where the minimum-phase and biproper plant $G(s)$ is given by

$$G(s) = \frac{(s + 3)(s + 7)}{(s - 1)(s + 5)}. \tag{2.21}$$

First $G(s)$ in (2.21) is factorized by (2.18), where

$$G_1(s) = \frac{s + 3}{s - 1} \tag{2.22}$$

and

$$G_2(s) = \frac{s + 7}{s + 5}. \tag{2.23}$$

Using $G_1(s)$ in (2.22), $Q(s) \in RH_\infty$ is given by (2.19) and written by

$$Q(s) = \frac{5(s + 0.6)}{(s + 3)(s + 1)}, \tag{2.24}$$

where k, α, and τ are settled by

$$k = 1, \tag{2.25}$$

$$\alpha = 1 \tag{2.26}$$

and

$$\tau = 1. \tag{2.27}$$

By substituting (2.19) into $(1 - Q(s))G(s)$, we obtain

$$(1 - Q(s))G(s) = \frac{s(s+7)}{(s+1)(s+5)}. \tag{2.28}$$

From (2.21) and (2.24), a stabilizing controller $C(s)$ written by the form of (2.3) to make the control system in (2.1) stable is written as

$$C(s) = \frac{5(s+0.6)(s+5)}{(s+1)(s+5)}. \tag{2.29}$$

$C(s)$ in (2.29) is obviously of minimum-phase. Therefore if $C(s)$ in (2.29) makes $G(s)$ in (2.21) stable, then $C(s)$ in (2.29) is a stabilizing minimum-phase controller for the minimum-phase plant $G(s)$ in (2.21).

Using the minimum-phase stabilizing controller $C(s)$ in (2.29), the response of the output $y(t)$ of the closed-loop system (2.1) for the step reference input $r(t) = 1$ is depicted in Figure I. Figure I verifies that the closed-loop system in (2.1) is stable and that the output $y(t)$ follows the step reference input $r(t) = 1$ with no steady-state error. In contrast, when the step disturbance $d(t) = 1$ is exerted, the response of the output $y(t)$ of the closed-loop system (2.1) is depicted in Figure II. Figure II verifies that the disturbance $d(t) = 1$ is completely attenuated.

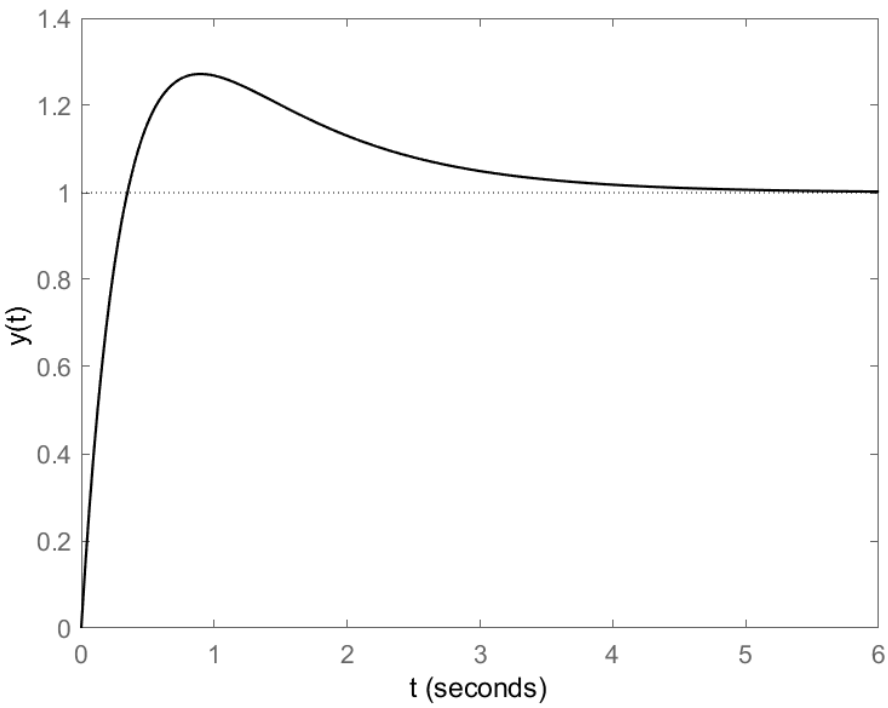

Fig. I: Response of the output $y(t)$ of the control system in (2.1) for the step reference input $r(t) = 1$

The presented example shows that the proposed method can design a minimum-phase stabilizing controller $C(s)$ based on reference tracking and the disturbance attenuation characteristics.

To check the robustness of the proposed design method, we consider the situation where a minimum-phase controller $C(s)$ in (2.29) is required to stabilize the plant

$$G_1(s) = \frac{(s-7.3)(s+1)}{(s-1)(s+3)(s+5)}, \tag{2.30}$$

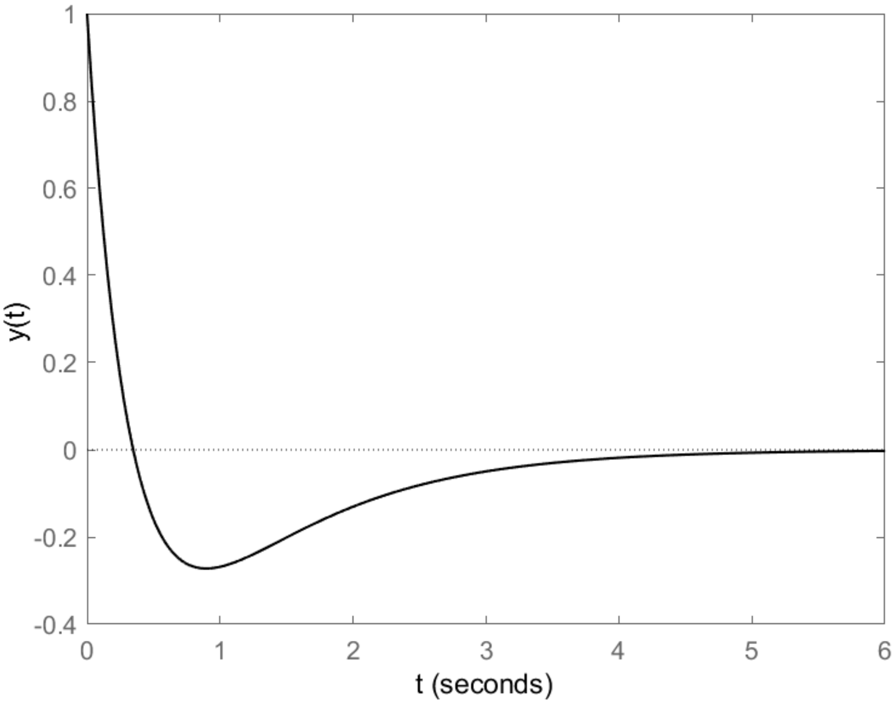

Fig. II: Response of the output $y(t)$ of the control system in (2.1) for the step disturbance $d(t) = 1$

which is obtained by perturbing the plant $G(s)$. In this situation, the response of the output $y(t)$ for the step reference input $r(t) = 1$ is depicted in Figure III. The response of the output $y(t)$ for the step disturbance $d(t) = 1$ is depicted in Figure IV. Figures III and IV indicate that the closed-loop system with the proposed controller (2.29) possesses robustness against the plant perturbation.

Furthermore, we compare the proposed design method with that of [5] where $C(s)$ is parametrized as

$$C(s) = \frac{1}{Q(s)} - \frac{1}{G(s)}. \tag{2.31}$$

By using the same $Q(s)$ in (2.24), we obtain another controller written as

$$C(s) = \frac{99(s+1)(s+3.602)(s^2-10.49s+35.9)}{(s+200)(s+7)(s+5)(s+2)}. \tag{2.32}$$

Here, the zeros of $C(s)$ are $-1, -3.602$ and $5.245 \pm 2.967j$, and $C(s)$ in (2.32) is obviously of nonminimum-phase. Therefore by using the method of [5], the controllers that we obtain are not necessarily of minimum-phase.

2.7 Conclusion

In this chapter, we clarified the parameterization of all stabilizing minimum-phase controllers for minimum-phase bipoper plants. That is, we showed that if the stabilizing controller $C(s)$ is written by the form of (2.3), the minimum-phase plant is stabilized. We also present a design method of the minimum-phase stabilizing controllers for the minimum-phase biproper plants that satisifies the robustness of the control system according to the internal model principle. In addition, we show a numerical example to illustrate the features of the proposed design method. In order to check the robustness of this example, we considered this situation that the minimum-phase controller stabilizes the perturbed plant. And we compare the proposed design method with that of [26]. By using the same $Q(s)$, the controller that we obtained by the proposed method is of minimum- phase. In contrast, by using the previous method, the controllers that we obtain are not necessarily of minimum-phase.

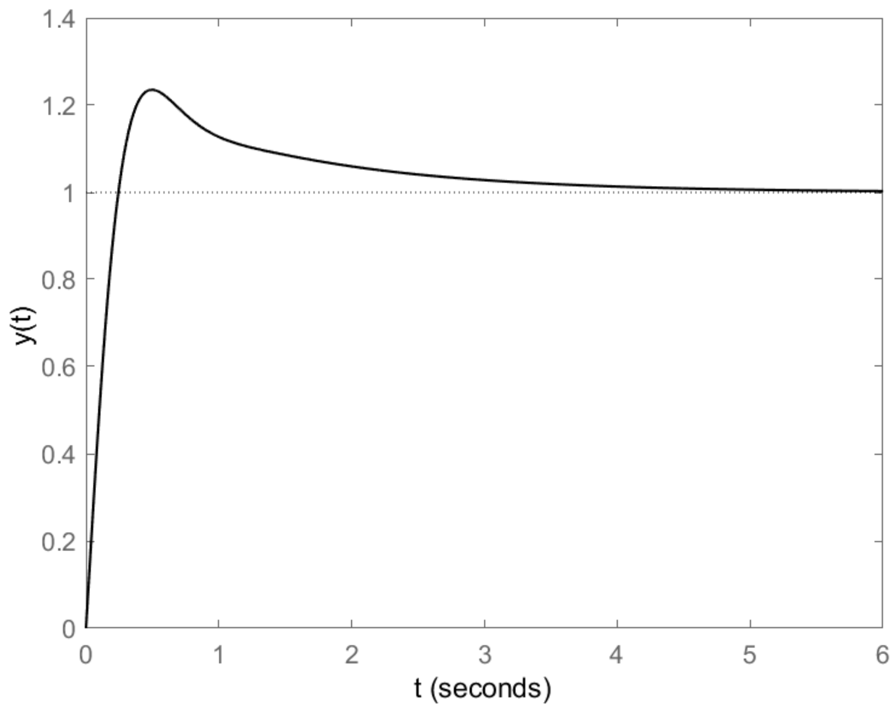

Fig. III: Response of the output $y(t)$ of the control system of $G_1(s)$ for the step reference input $r(t) = 1$

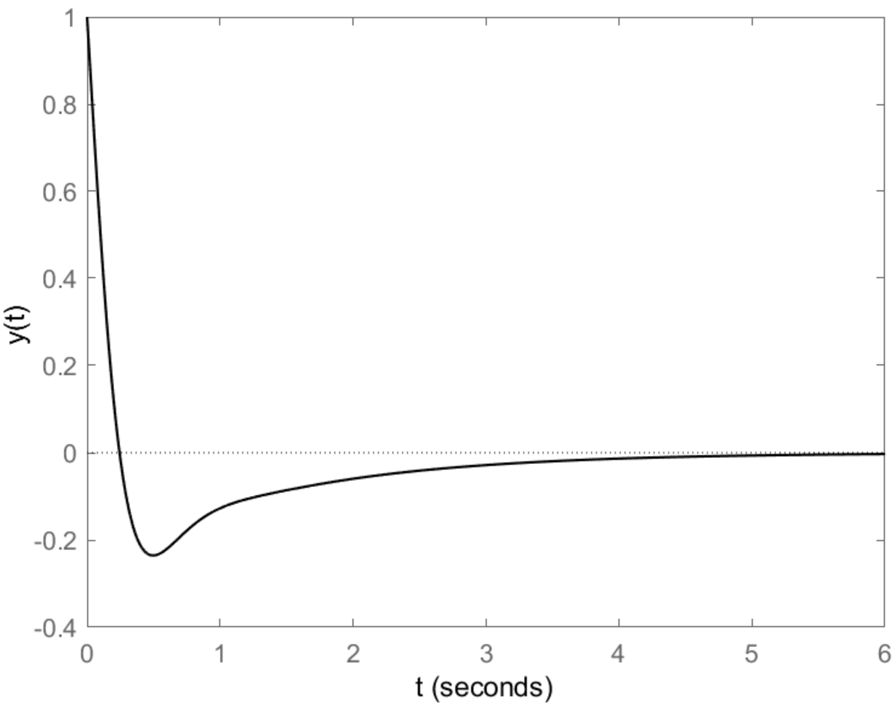

Fig. IV: Response of the output $y(t)$ of the control system of $G_1(s)$ for the step disturbance $d(t) = 1$

Chapter 3

Minimum-Phase Controllers for Minimum-Phase SISO Strictly Proper Plants

3.1 Introduction.

In this chapter, we expend the parameterization of all stabilizing minimum-phase controllers for minimum-phase biproper single-input single-output plants to the parameterization of all stabilizing minimum-phase controllers for minimum-phase strictly proper single-input single-output plants. In the minimum-phase single-input single-output system, the parameterization of all stabilizing minimum-phase controllers is given.

Glaria and Goodwin provided a simplified parameterization for the stability control of the minimum-phase[4]. However, there are two difficult problems. One of them is that the parameterization of stabilizing controllers proposed by Glaria and Goodwin usually contain improper controllers. In the specific process of use, a proper controller is needed. Second, the internally stability in the system is not parameterized. In order to solve the above problems, Yamada presents a parameterization for the class of all proper stabilizing controllers for linear minimum phase systems [5]. In order to guarantee proper controllers, a restriction of the free parameter $Q(s)$ by Glaria and Goodwin is traded by a choice of an adequate rational function $K(s)$. It is shown that there exists an adequate rational function $K(s)$. To achieve the internally stability condition, it is shown that the free parameter $Q(s)$ must be biproper stable rational function.

As the parameterization of the minimum-phase stabilizing controllers for the minimum-phase biproper plants is proposed, the parameterization of the minimum-phase stabilizing controllers for the minimum-phase strictly proper plants is required. Since the parameterization of the minimum-phase stabilizing controllers for the minimum-phase plants is clarified, it is used in a wide range of applications, for example it can be used as a special inverse system for strong stable control, which is stable for both the controller and the controlled plant [29]. If the stabilizing controller with nonminimum-phase is used, its unstable zeros will cause the transfer function of the closed-loop system to have zeros on its right half plane. This makes the closed-loop control system very sensitive to the disturbance of the external environment, thus affecting the control effect. In addition, if the feedback loop is truncated, that is, it is split into a feedforward, then the instability caused by it will lead to instability [7, 8]. In this way, even though the controlled plant is of minimum-phase, the control system becomes an non-minimum system. If the minimum-phase control is adopted, the target of the minimum-phase will remain unchanged, and the the magnitude of sensitivity of the whole system will become small. The lower the sensitivity curve, the greater the damping to external interference. If the minimum-phase controllers of the minimum-phase plants can be parameterized, a new control strategy for the minimumphase system can be obtained. Therefore, for the strictly proper controlled plants with minimum-phase, the minimum-phase controllers must be parameterized.

In this chapter, we propose the parameterization of all stabilizing minimum-phase controllers for minimum-phase strictly proper plants. That is, we consider the parameterization that the stabilizing controller makes minimum-phase plant stable, which the stabilizing controller is of minimum-phase. Analysis of the internal stability and control characteristics of closed-loop system are provided. We also present a design method of the minimum-phase stabilizing controllers that contributes to the construction of a minimum-phase closed-loop system. In addition, we show a numerical example to illustrate the features of the proposed design method. This paper is organized as follows: In Section 3.2 , we show the problem considered in this chapter. In Section 3.3 , we clarify the parameterization of all stabilizing minimum-phase controllers for minimum-phase strictly proper singleinput singleoutput plants and the analysis the internal stability of closed-loop system. In Section 3.4 , we present the control system characteristics with the parameterization of all stabilizing minimum-phase controllers for minimum-phase strictly proper plants. In Section 3.5 , we present a design method of stabilizing minimum-phase controllers based on the parameterization of all stabilizing minimum-phase controllers for minimum-phase plants given in Section 3.3. In Section 3.6 , we show a numerical example to illustrate the effectiveness of the proposed method. And check the robustness of this example by using a perturbed plant. In Section 3.7 , we give concluding remarks.

3.2 Problem formulation

In this section, we introduce the problem considered in this paper. We consider a closed loop feedback control system as,

$$\left\{ \begin{array}{rcl} y(s) & = & G(s)u(s) + d(s) \\ u(s) & = & C(s)\left(r(s) - y(s)\right) \end{array} \right. . \tag{3.1}$$

Here, $y(s) \in R(s)$, $u(s) \in R(s)$, $r(s) \in R(s)$ and $d(s) \in R(s)$ are the output, control input, reference input and disburtance of the control system respectively. $G(s) \in R(s)$ and $C(s) \in R(s)$ are the controller and the plant of the control system separately, and both of them are of minimum-phase, that means, all zeros of them are in the left half plane. In this paper, the controlled plant $G(s)$ with minimum-phase is required to be strictly proper and it is possible to be stable or unstable. By using the parameterization of the minimum-phase stabilizing controller for the minimum-phase plant, the controller $C(s)$ is required to be derived. In the specific process of use, a proper controller is needed and the internally stability and the robustness of the control system need to be considered. Here $R(s)$ indicates the set of real rational functions for the set with s.

The problem considered in this paper is to clarify the parameterization of all stabilizing minimum-phase controllers $C(s)$ for minimum-phase strictly proper plants $G(s)$.

3.3 The Parameterization

In this section, we clarify the parameterization of all stabilizing minimum-phase controllers $C(s)$ for minimum-phase strictly proper plants $G(s)$.

This parameterization is summarized in the following theorem.

Theorem 3.3.1 *Assume that $G(s)$ is strictly proper and of minimum-phase. When $K(s)$ exists in a system of equations, $K(s)$ is a stable and biproper real rational function and make $G(s)+K(s)$ be a biproper and minimum-phase real rational function. Utilizing the above $K(s)$, for all proper minimum-phase stabilizing controllers $C(s)$ of the plant $G(s)$ with strictly proper and minimum-phase, the parameters are as follows*

$$C(s) = \frac{\bar{C}(s)}{1 + \bar{C}(s)K(s)}. \tag{3.2}$$

Here, $\bar{C}(s)$ is expressed as

$$\bar{C}(s) = \frac{\bar{Q}(s)}{(1 - \bar{Q}(s))(G(s) + K(s))} \tag{3.3}$$

and $\bar{Q}(s)$ is the minimum-phase function belong to RH_∞ and to make $(1 - \bar{Q}(s))(G(s) + K(s)) \in RH_\infty$.

For proving the Theorem 3.3.1, the following thereoms are needed.

Theorem 3.3.2 *Assume that $G(s)$ is strictly proper and minimum-phase. When $K(s)$ exists in a system of equations, $K(s)$ is a stable and biproper real rational function and make $G(s) + K(s)$ be a biproper and minimum-phase real rational function.*

(Proof)
At first, $G(s)$ is factorized into the coprime factors with $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ on RH_∞ and $G(s)$ is rewriten in the form of

$$G(s) = \frac{N(s)}{D(s)}. \tag{3.4}$$

Here, because $G(s)$ is assumed to be of minimum-phase and strictly proper, $N(s) \in RH_\infty$ is of minimum-phase and strictly proper. In addition, $G(s) + K(s)$ is donated as

$$G(s) + K(s) = \frac{N(s) + K(s)D(s)}{D(s)}, \tag{3.5}$$

Then, the exist condition of $K(s)$ is that $G(s)+K(s)$ is a biproper and minimum-phase, and consistent with that of $U(s) \in \mathcal{U}$ and $K(s) \in RH_\infty$ satisfying

$$U(s) = N(s) + K(s)D(s) \in \mathcal{U}. \tag{3.6}$$

Here, $\mathcal{U}$ is the set of unimodular functions on RH_∞, so $U(s) \in \mathcal{U}$ implies $U(s) \in RH_\infty$ and $U^{-1}(s) \in RH_\infty$. The existence conditions of $U(s)$ and $K(s)$ is equivalent to the interpolation problem and are written as

$$\frac{d^j}{ds^j}U(s_i) = \frac{d^j}{ds^j}N(s_i) \quad (j = 0, ...m_i - 1; i = 1, ...l), \tag{3.7}$$

where $s_1, \ldots, s_l$ are different zeros of $D(s)$ on the positive real axis, $m_1, \ldots, m_l$ are the corresponding multiplicities and l denotes the number of different zeros of $D(s)$ on the positive real axis. Since $G(s)$ is of minimum-phase, $N(s)$ is also of minimum-phase. This implies that all of $N(s_i)$ are the same sign. From Theorem 2.3.1 in [7], there exists $U(s) \in \mathcal{U}$ and $K(s) \in RH_\infty$ satisfying (3.7). This implies that there exists $U(s) \in \mathcal{U}$ and $K(s) \in RH_\infty$ satisfying (3.6).

The remaining problem is whether or not, $K(s)$ is biproper. Next, it is shown that if $U(s) \in \mathcal{U}$ exists such that (3.6) holds true, then $K(s)$ is biproper. From (3.6), $K(s)$ is written by

$$K(s) = \frac{U(s) - N(s)}{D(s)}. \tag{3.8}$$

The assumption that $U(s)$ holds (3.6) implies that $K(s)$ written by (3.8) is stable. Since both $U(s)$ and $D(s)$ are biproper and $N(s)$ is strictly proper, $K(s)$ denoted by (3.8) is biproper.

We have thus proved the theorem.

Theorem 3.3.3 *Assume that $\bar{G}(s) = G(s)+K(s)$ is the real rational function with strictly proper and minimum-phase. All minimum-phase stabilizing controllers $\bar{C}(s)$, its parameterization for the plant $\bar{G}(s)$ is denoted as*

$$\bar{C}(s) = \frac{\bar{Q}(s)}{(1-\bar{Q}(s))\bar{G}(s)}. \tag{3.9}$$

Here, $\bar{Q}(s) \in RH_\infty$ is any minimum-phase function and to make $(1-\bar{Q}(s))\bar{G}(s) \in RH_\infty$.

(Proof)

Because $\bar{G}(s) = G(s) + K(s)$ is assumed to be the real rational function with strictly proper and minimum-phase, according to Thereom 2.3.1, the minimum-phase stabilizing controllers $\bar{C}(s)$, its parameterization for $\bar{G}(s)$ is denoted as (3.9).

We have thus proved the theorem.

Theorem 3.3.4 *Assume that $K(s) \in RH_\infty$ is biproper and $G(s)$ is strictly proper. If the minimum-phase controller $C(s)$ stabilizes the plant $G(s)$, then $\bar{C}(s)$ is written as*

$$\bar{C}(s) = \frac{C(s)}{1 - C(s)K(s)} \tag{3.10}$$

stabilizes the plant $\bar{G}(s) = G(s) + K(s)$. Furthermore, the opposite is also true. That is, if the minimum-phase controller $\bar{C}(s)$ stabilizes the plant $\bar{G}(s) = G(s) + K(s)$, then the the minimum-phase controller $C(s)$ is written as

$$C(s) = \frac{\bar{C}(s)}{1 + \bar{C}(s)K(s)} \tag{3.11}$$

stabilizes the plant $G(s)$.

(Proof)

First, we will prove that if the minimum-phase controller $C(s)$ stabilizes $G(s)$, then the minimum-phase controller $\bar{C}(s)$ written by (3.10) stabilizes $\bar{G}(s) = G(s) + K(s)$. $K(s)$ is assumed to be biproper and $C(s)$ is assumed to be of minimum-phase. In (3.10) if the $(1 - C(s)K(s))^{-1}$ has unstable zeros, the unstable zeros are the unstable poles of $C(s)$. Therefore, the $\bar{C}(s)$ has no unstable zeros, that is, $\bar{C}(s)$ is of minimum-phase. Then from (3.10) and simple manipulation, $1/(1+\bar{C}(s)\bar{G}(s))$, $\bar{C}(s)/(1+\bar{C}(s)G(s))$, $\bar{G}(s)/(1+\bar{C}(s)\bar{G}(s))$ and $\bar{C}(s)\bar{G}(s)/(1+\bar{C}(s)\bar{G}(s))$ are rewritten as

$$\frac{1}{1+\bar{C}(s)\bar{G}(s)} = \frac{1 - C(s)K(s)}{1 + G(s)C(s)}, \tag{3.12}$$

$$\frac{\bar{C}(s)}{1+\bar{C}(s)\bar{G}(s)} = \frac{C(s)}{1+G(s)C(s)}, \tag{3.13}$$

$$\frac{\bar{G}(s)}{1+\bar{C}(s)\bar{G}(s)} = \frac{(G(s)+K(s))(1-C(s)K(s))}{1+G(s)C(s)}, \tag{3.14}$$

and

$$\frac{\bar{C}(s)\bar{G}(s)}{1+\bar{C}(s)\bar{G}(s)} = \frac{(G(s)+K(s))C(s)}{1+G(s)C(s)}. \tag{3.15}$$

From the assumption that $C(s)$ stabilizes $G(s)$, $1/(1+C(s)G(s))$, $C(s)/(1+C(s)G(s))$, $G(s)/(1+C(s)G(s))$ and $C(s)G(s)/(1+C(s)G(s))$ are all include in RH_∞. Therefore, all of transfer functions in (3.12), (3.13), (3.14) and (3.15) are include in RH_∞.

Next, we will show that if the minimum-phase controller $\bar{C}(s)$ stabilizes the plant $\bar{G}(s) = G(s)+K(s)$, then the minimum-phase controller $C(s)$ written by (3.11) stabilizes $G(s)$. In (3.11) if the $(1+\bar{C}(s)K(s))^{-1}$ has unstable zeros, the unstable zeros are the unstable poles of $C(s)$. Therefore, the $C(s)$ has no unstable zeros, that is, $C(s)$ is of minimum-phase. Then from (3.11) and simple manipulation, $1/(1+C(s)G(s))$, $C(s)/(1+C(s)G(s))$, $G(s)/(1+C(s)G(s))$ and $C(s)G(s)/(1+C(s)G(s))$ are rewritten as

$$\frac{1}{1+C(s)G(s)} = \frac{1+\bar{C}(s)K(s)}{1+\bar{G}(s)\bar{C}(s)}, \tag{3.16}$$

$$\frac{C(s)}{1+C(s)G(s)} = \frac{\bar{C}(s)}{1+\bar{G}(s)\bar{C}(s)}, \tag{3.17}$$

$$\frac{G(s)}{1+C(s)G(s)} = \frac{(\bar{G}(s)-K(s))(1+\bar{C}(s)K(s)}{1+\bar{G}(s)\bar{C}(s)}, \tag{3.18}$$

and

$$\frac{C(s)G(s)}{1+C(s)G(s)} = \frac{(\bar{G}(s)-K(s))\bar{C}(s)}{1+\bar{G}(s)\bar{C}(s)}. \tag{3.19}$$

From the assumption that $\bar{C}(s)$ stabilizes $\bar{G}(s) = G(s)+K(s)$, $1/(1+\bar{C}(s)\bar{G}(s))$, $\bar{C}(s)/(1+\bar{C}(s)\bar{G}(s))$, $\bar{G}(s)/(1+\bar{C}(s)\bar{G}(s))$ and $\bar{C}(s)\bar{G}(s)/(1+\bar{C}(s)\bar{G}(s))$ are all include in RH_∞. Therefore, all of transfer functions in (3.16), (3.17), (3.18) and (3.19) are include in RH_∞.

We have thus proved Theorem 3.3.4.

Theorem 3.3.1 is proved using the above-described theorems.

(Proof)

From Theorem 3.3.2, there exists biproper $K(s) \in RH_\infty$ to make $\bar{G}(s) = G(s)+K(s)$ of minimum phase. From Theorem 3.3.4, the parametrization of all internally stabilizing controllers $C(s)$ for $G(s)$ is same to that of all internally stabilizing controllers $\bar{C}(s)$ for $\bar{G}(s) = G(s)+K(s)$. The parametrization of all internally stabilizing controllers $\bar{C}(s)$ for $\bar{G}(s) = G(s)+K(s)$ is given by (3.9), where $\bar{Q}(s) \in RH_\infty$ is any minimum-phase function to make $(1-\bar{Q}(s))\bar{G}(s) \in RH_\infty$. The equation (3.9) corresponds to (3.3). From Theorem 3.3.4, using $\bar{C}(s)$, $C(s)$ is written in terms of (3.11). The equation (3.11) corresponds to (3.2). The proof of Theorem 3.3.1 is complete.

3.4 Characteristics of the control system

We elucidate the properties of the closed-loop control system using the parameterization of all the stabilizing minimum-phase controllers given by (3.2) in this section.

First, we consider the reference tracking property. Here, using the parameterization of all minimum-phase stabilizing controllers for the minimum-phase plants in (3.2), the transfer function in (3.1) from the reference input $r(s)$ to the output $y(s)$ of the control system is given as

$$\frac{y(s)}{r(s)} = \frac{\bar{Q}(s)G(s)}{G(s)+K(s)}. \tag{3.20}$$

Therefore, in order to make the output $y(s)$ follow the step reference input $r(s) = 1/s$ without steady-state error,

$$\frac{\bar{Q}(0)G(0)}{G(0)+K(0)} = 1 \tag{3.21}$$

must be achieved.Thus, the output $y(s)$ follows the step reference input $r(s) = 1/s$ without steady-state error, if $\bar{Q}(s)$ satisfies the following condition

$$\bar{Q}(0) = 1 + \frac{K(0)}{G(0)}. \tag{3.22}$$

Next, the decay properties of the disturbance are described. The transfer function from the disturbance $d(s)$ to the output $y(s)$ is given as

$$\frac{y(s)}{d(s)} = 1 - \frac{\bar{Q}(s)G(s)}{G(s) + K(s)}. \tag{3.23}$$

Therefore, in order to fully decay the step disturbance $d(s) = 1/s$,

$$\frac{\bar{Q}(0)G(0)}{G(0) + K(0)} = 1 \tag{3.24}$$

must be achieved. Thus, the step disturbance of $d(s) = 1/s$ will effectively be rejected if $\bar{Q}(s)$ satisfies the following condition

$$\bar{Q}(0) = 1 + \frac{K(0)}{G(0)}. \tag{3.25}$$

3.5 Design method of minimum-phase stabilizing controllers

First, the concept of the design is considered. In this chapter, the plant $G(s)$ is of minimum-phase and stricrly proper. The parameterization for biproper minimum-phase plants can not be used for the strictly proper plants, because that will make the controller to be not proper. From Theorem 3.3.2, $K(s)$ is stable and biproper and to make the $G(s) + K(s)$ be of minimum-phase and biproper. Then, $\bar{Q}(s) \in RH_\infty$ is determined as a function of any minimum-phase such that $(1 - \bar{Q}(s))(G(s) + K(s)) \in RH_\infty$ from Theorem 3.3.3. Furthermore, in order to make the output $y(s)$ follow the reference input $r(s) = 1/s$ without steady-state error and attenuate the step disturbance $d(s) = 1/s$ completely, $\bar{Q}(s)$ needs to satisfy (3.22). Then a design method for the minimum-phase stabilizing controller $C(s)$ is concluded as follows.

1. Obtain $K(s)$ that satisfies Theorem 3.3.2, and such that $G(s) + K(s)$ is of minimum-phase and biproper.
2. Obtain $\bar{G}(s) = G(s) + K(s)$ that is of minimum-phase and biproper function.
3. Define a function $\hat{Q}(s)$ as

$$\hat{Q}(s) = -\frac{K(s)}{G(s)}. \tag{3.26}$$

4. Using $\hat{Q}(s)$ in (3.26), design $\bar{Q}(s) \in RH_\infty$ as

$$\bar{Q}(s) = 1 - \hat{Q}(s)\frac{1}{(\tau s + 1)^\alpha}, \tag{3.27}$$

   where $\tau \in R$ and α is an arbitrary positive integer to make $\bar{Q}(s)$ proper, that is α is greater than or equal to the relative degree of the plant $G(s)$.

5. Using $\bar{Q}(s)$ in (3.27), fix a minimum-phase stabilizing controller $C(s)$ in (3.2).

First, we show the parameter $\bar{Q}(s) \in RH_\infty$ is of minimum-phase. Even if $K(s)$ is biproper and $G(s)$ is strictly proper,there exists α that is greater than or equal to the relative degree of the plant $G(s)$, in order that $\bar{Q}(s)$ is proper in (3.27). In (3.26), since $K(s)$ is stable and $G(s)$ is of minimum-phase, $\hat{Q}(s)$ is stable. Substituting $\hat{Q}(s)$ in (3.26) into (3.27), $\bar{Q}(s)$ is stable an of minimum-phase. Next, we check that $(1 - \bar{Q}(s))(G(s) + K(s)$ is stable and proper. By substituting (3.27) and (3.4) into $(1 - \bar{Q}(s))(G(s) + K(s)$,

$$(1 - \bar{Q}(s))(G(s) + K(s) = -\frac{1}{(\tau s + 1)^\alpha}\frac{K(s)}{N(s)}(N(s) + K(s)D(s)). \tag{3.28}$$

Here, $N(s)+K(s)D(s)$ belong to $\mathcal{U}$, $K(s)$ is biproper and stable and $N(s)$ is strictly proper and minimum-phase, so $(1 - \bar{Q}(s))(G(s) + K(s))$ is proper and stable. Finally, we find that $\bar{Q}(s)$ defined by (3.27) is guaranteed to satisify (3.22) and (3.25), that means, the proposed method can design a minimum-phase stabilizing controller $C(s)$ based on reference tracking and the disturbance attenuation characteristics.

3.6 Numerical example

This section shows a numerical example to illustrate the features of the proposed design method.

Consider the problem to find a minimum-phase stabilizing controllers for the plant $G(s)$ written by

$$G(s) = \frac{(s+3)}{(s-1)(s+2)}. \tag{3.29}$$

By Thereom 3.3.2, we obtian

$$K(s) = \frac{(s+1.657)(s+3.529)(s+5.814)}{(s+2)^2(s+9)}. \tag{3.30}$$

Then, $\bar{G}$ is writen as

$$\bar{G}(s) = G(s) + K(s) = \frac{(s+5)(s+4)(s+1)^2}{(s-1)(s+2)^2(s+9)}. \tag{3.31}$$

From (3.26), $\hat{Q}(s)$ is written by

$$\hat{Q}(s) = -\frac{(s+1.657)(s+3.529)(s+5.814)(s-1)}{(s+9)(s+3)(s+2)}. \tag{3.32}$$

$\bar{Q}(s)$ is given by (3.27), where α, and τ are settled by

$$\alpha = 1 \tag{3.33}$$

$$\tau = 0.1, \tag{3.34}$$

Then $\bar{Q}(s)$ is obtained as

$$\bar{Q}(s) = \frac{11(s+5.542)(s+3.735)(s+1.34)(s+0.6555)}{(s+9)(s+10)(s+3)(s+2)}. \tag{3.35}$$

Then the minimum-phase stabilizing controller $C(s)$ is obtained as

$$C(s) = \frac{11(s+5.542)(s+3.735)(s+2)(s+1.34)(s+0.6555)}{s(s+3)(s+3.529)(s+1.657)(s+5.814)}. \tag{3.36}$$

For the step reference input $r(t) = 1$, the output of the closed-loop system $y(t)$ reacts as follows I when using the minimum-phase stabilizing controller $C(s)$ (3.36). In the curve of Figure I, it is proved that the controlled system is stable under the equation (3.1), and its output $y(t)$ is equal to the step reference input $r(t) = 1$, and there is no steady-state deviation.

On the contrary, if there is a step disturbance, the output response of the closed-loop is displayed in the Figure II. The curve in Figure II confirms which the effect of this interference on $d(t) = 1$ is effective.

Next, to verify the robustness of the proposed method, we consider this case where we control a perturbed controlled plant with the controller that has been derived. The controlled plant is denoted as

$$G_1(s) = \frac{(s+10)}{(s-1)(s+2)}. \tag{3.37}$$

In this case, the output response corresponding to the step input $r(t) = 1$ is represented in Figure III. The figure reflects that output y(t) follows the step reference input $r(t)$ well with no steady state error. And the output response corresponding to the step disturbance $d(t) = 1$ is represented in Figure IV. The figure reflects the ability to effectively stop external disturbance.

Furthermore, we compare the proposed design method with that of [5] where the controller $C(s)$ for the strictly proper plant is parametrized as

$$C(s) = \frac{\bar{C}(s)}{1 + \bar{C}(s)K(s)}. \tag{3.38}$$

Here, $\bar{C}(s)$ is expressed as

$$\bar{C}(s) = \frac{1}{\bar{Q}(s)} - \frac{1}{G(s) + K(s)}. \tag{3.39}$$

By using the same $Q(s)$ in (3.35) and $K(s)$ in (3.30), we obtain another controller written as

$$C(s) = \frac{-10(s-2.358)(s+2.192)(s+2)^2(s+1.254)(s+0.7462)(s+3.744)(s+5.522)(s+9)}{(s+18.74)(s+8.692)(s+5.533)(s+3.741)(s+3)(s+2.56)(s+1.733)(s+1.258)(s+0.744)}. \tag{3.40}$$

Here, one the zeros of $C(s)$ is 2.358 and $C(s)$ in (3.40) is obviously of nonminimum-phase. Therefore by using the method of [5], the controllers that we obtain are not necessarily of minimum-phase.

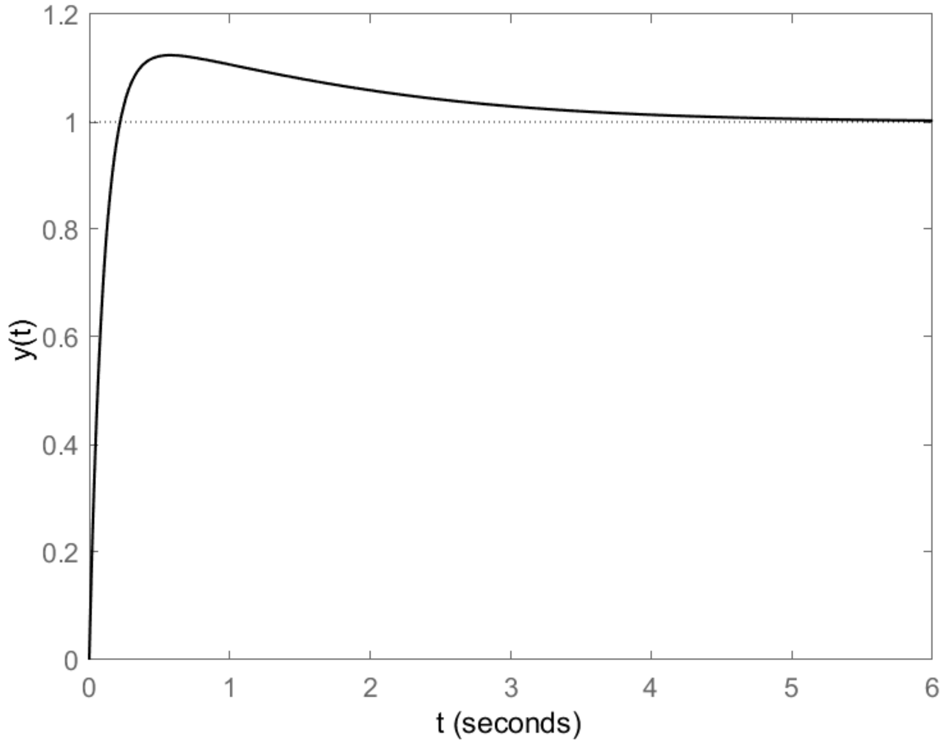

Fig. I: the response of the output of $y(t)$ of the controlled system corresponding to the step reference input $r(t) = 1$

3.7 Conclusions.

In this chapter, we propose the parameterization of all minimum-phase stabilizing controllers for minimum-phase stabilizing strictly proper single-input single-output plants. Analysis of the internal stability and control characteristics of closed-loop system are provided. We also present the control system characteristics with the parameterization of all stabilizing minimum-phase controllers for minimum-phase strictly proper plants. The reference tracking characteristic and the disturbance attenuation characteristic is considered. In additon, a numerical example is illustrated for the effectiveness of the proposed method and the robustness of this example is checked by using a perturbed plant.

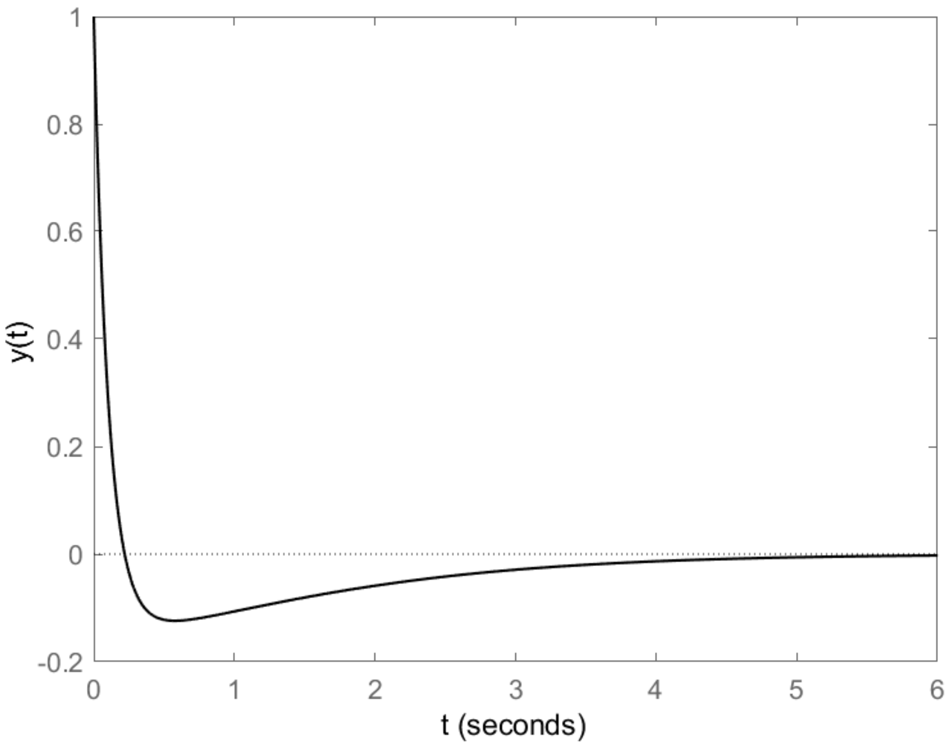

Fig. II: the response of the output of $y(t)$ of the controlled system corresponding to the step disturbance $d(t) = 1$

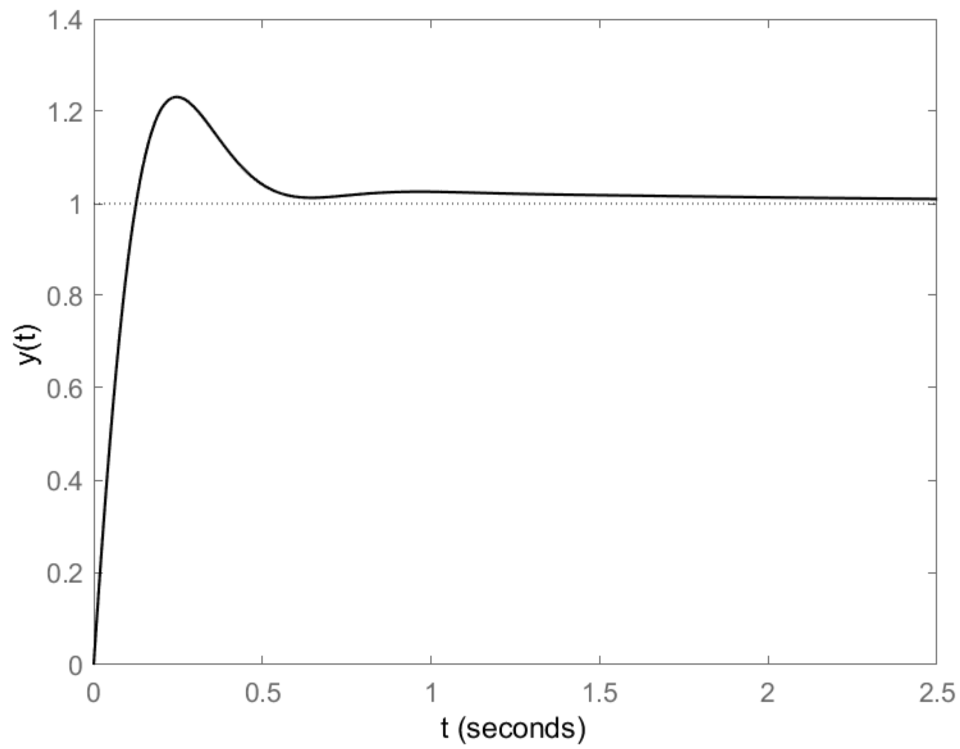

Fig. III: the response of the output of $y(t)$ of the controlled system with G_1 corresponding to the sstep reference input $r(t) = 1$

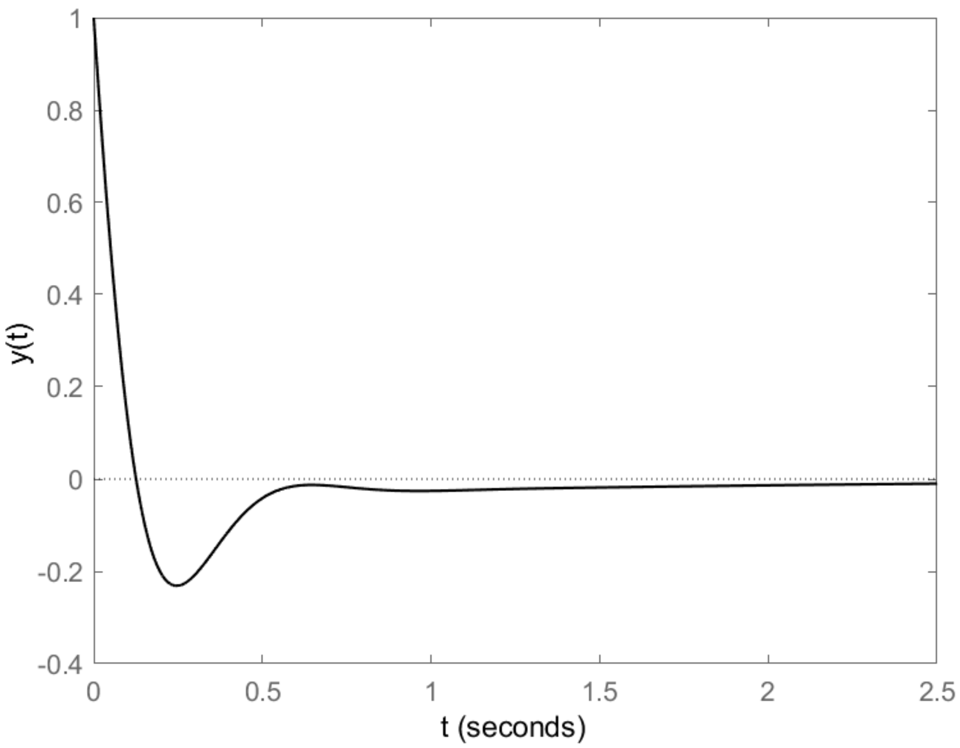

Fig. IV: the response of the output of $y(t)$ of the controlled system with G_1 corresponding to the step disturbance $d(t) = 1$

Chapter 4

Minimum-Phase Controllers for Minimum-Phase MIMO Plants

4.1 Introduction.

In the chapter, we examine the parameterization of all stabilizing minimum-phase controllers for minimum-phase multiple-input/multiple-output plants. The parameterization problem is the problem in which all stabilizing controllers for plants are sought. Since this parameterization can successfully search for all stabilizing minimum-phase controllers, it is used as a tool for many control problems.

Yamada presents a parameterization for the class of all proper stabilizing controllers for linear minimum phase systems [5]. The parameterization of all stabilizing controllers in [5] is applied to many control problems such as the parameterization of all stabilizing modified repetitive controllers for minimum-phase plants [20], adaptive control systems [21, 22], model feedback control systems [23], parallel compensation technique [24], PI control [25] and PID control [26]. In [28], Yamada et al. expand the result in [5] and propose the parametrization of all stabilizing controllers for multiple-input multiple-output minimum phase systems. As the parametrization of all stabilizing controllers for multiple-input/multiple-output minimum phase systems is obtained, the results in [20, 21, 22, 23, 24, 25, 26] are expanded for multiple-input multiple-output minimum-phase systems.

If a nonminimum-phase stabilizing controller is employed, the unstable zeros of the stabilizing controller make the closed-loop transfer function have zeros in the right half plane. This results in the closed-loop system that is very sensitive to disturbances and reduces the tracking performance to the reference input. Using the minimum-phase controllers can make the sensitivity function lower and the lower values of sensitivity function suggest further attenuation of the external disturbance. From this viewpoint, it is desirable to clarify the parameterization of all stabilizing minimum-phase controllers for minimum-phase multiple-mnput/multiple-output plants. The purpose of this paper is to propose the parameterization of all stabilizing minimum-phase controllers for minimum-phase multiple-input/multiple-output plants.

This chapter is organized as follows: In Section 4.2 , we show the problem considered in this chapter. In Section 4.3 , we clarify the parameterization of all stabilizing minimum-phase controllers for minimum-phase biproper multiple-input multiple-output plants and the analysis the internal stability of closed-loop system. In Section 4.4 , we show the control system characteristics with the parameterization of all stabilizing minimum-phase controllers for minimum-phase biproper multiple-input multiple-output plants. In Section 4.5 , we propose a design method of stabilizing minimum-phase controllers based on the parameterization of all stabilizing minimum-phase controllers for minimum-phase plants given in Section 4.3 satisifying the robustness. In Section 4.6 , we show a numerical example to illustrate the features of the proposed design method. In Section 4.7 , we clarify the parameterization of all stabilizing minimum-phase controllers for minimum-phase strictly proper multiple-input multiple-output plants. In Section 4.8 , we give concluding remarks.

4.2 Problem Formulation

Consider the control system in

$$\left\{ \begin{array}{rcl} y(s) & = & G(s)u(s) + d(s) \\ u(s) & = & C(s)(r(s) - y(s)) \end{array} \right. , \tag{4.1}$$

where $G(s) \in R^{m \times m}(s)$ is the plant, $C(s) \in R^{m \times m}(s)$ is the controller, $y(s) \in R^m(s)$ is the output , $u(s) \in R^m(s)$ is the control input and $r(s) \in R^m(s)$ is the reference input. $R(s)$ denotes the set of real rational functions with s. $G(s)$ and $C(s)$ are assumed to be of minimum-phase, that is, $G(s)$ and $C(s)$ have no zero in the closed right half plane. And $G(s)$ is assumed to be biproper or strictly proper, that is,

$$\det \lim_{s\to\infty} \{G(s)\} \neq 0 \tag{4.2}$$

or

$$\det \lim_{s\to\infty} \{G(s)\} = 0 \tag{4.3}$$

hold ture separately. The minimum-phase controller controls the minimum-phase plant to make the closed-loop system in (4.1) stable.

The problem considered in this paper is to clarify the parameterization of all stabilizing minimum-phase controllers for minimum-phase MIMO plants.

4.3 The Parameterization

In this section, we clarify the parameterization of all stabilizing minimum-phase controllers $C(s)$ for minimum-phase MIMO biproper plants $G(s)$.

This parameterization is summarized in the following theorem.

Theorem 4.3.1 *$G(s)$ is assumed to be of minimum-phase and to be biproper. Then the minimum-phase controller $C(s)$ stabilizes the feedback control system in (4.1) if and only if $C(s)$ is written by the form of*

$$C(s) = ((I - Q(s))G(s))^{-1} Q(s)$$
$$\left(\det\{\lim_{\omega\to\infty} ((I - Q(j\omega))G(j\omega))\} \neq 0\right), \tag{4.4}$$

where $Q(s) \in RH_\infty^{m\times m}$ is any minimum-phase function to make $(I - Q(s))G(s) \in RH_\infty$.

(Proof)

First, the necessity is shown. That is, we show that if the minimum-phase controller $C(s)$ makes minimum-phase plant $G(s)$ stable, then $C(s)$ takes the form of (4.4). From the assumption that $C(s)$ in (4.4) makes $G(s)$ in (4.1) stable, $(I+G(s)C(s))^{-1}$, $(I+G(s)C(s))^{-1}G(s)$, $C(s)(I+G(s)C(s))^{-1}$ and $(I+G(s)C(s))^{-1}G(s)C(s)$ are all included in RH_∞. RH_∞ denotes the set of stable proper real rational functions.

From the assumption that $G(s)$ and $C(s)$ are both assumed to be of minimum-phase,

$$(G(s)C(s))^{-1}(I + G(s)C(s)) = I + (G(s)C(s))^{-1} \tag{4.5}$$

is stable. Using $Q(s) \in RH_\infty$, $((I + G(s)C(s))^{-1}G(s)C(s) \in RH_\infty$ can be rewritten as

$$(I + G(s)C(s))^{-1}G(s)C(s) = Q(s). \tag{4.6}$$

From (4.5) and (4.6), $Q(s)$ must be of minimum-phase. From simple manipulation, (4.6) is rewritten as

$$C(s) = ((I - Q(s))G(s))^{-1} Q(s), \tag{4.7}$$

Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, if $C(s)$ takes the form of (4.4), then the minimum-phase controller $C(s)$ stabilizes the minimum-phase plant $G(s)$ to make the control system stable. Then we set $C(s)$ as

$$C(s) = ((I - Q(s))G(s))^{-1} Q(s), \tag{4.8}$$

where $Q(s) \in RH_\infty$ is any minimum-phase function and $(1-Q(s))G(s) \in RH_\infty$ is any function. If the controller $C(s)$ makes $G(s)$ stable, according to definition of internal stability, the transfer functions $(I+G(s)C(s))^{-1}$, $(I+G(s)C(s))^{-1}G(s)$, $C(s)(I+G(s)C(s))^{-1}$ and $(I+G(s)C(s))^{-1}G(s)C(s)$ are stable. After simple manipulation, the transfer functions are rewritten as

$$(I + G(s)C(s))^{-1} = I - Q(s), \tag{4.9}$$

$$C(s)(I + G(s)C(s))^{-1} = Q(s)(G(s))^{-1}, \tag{4.10}$$

$$(I + G(s)C(s))^{-1}G(s) = (1 - Q(s))G(s) \tag{4.11}$$

and

$$(I + G(s)C(s))^{-1}G(s)C(s) = Q(s). \tag{4.12}$$

Because $Q(s) \in RH_\infty$ and $G(s)$ is of minimum-phase, transfer functions in (4.9), (4.10) and (4.12) are stable. If the transfer function in (4.11) is unstable, unstable poles of the transfer function in (4.11) are unstable poles of $G(s)$. From the assumption that $(I-Q(s))G(s) \in RH_\infty$, unstable poles of $G(s)$ are not poles of $(I-Q(s))G(s)$. Therefore, the transfer function in (4.11) is stable. Thus, the sufficiency has been shown.

We have thus proved Theorem 4.3.1.

4.4 Characteristics of closed-loop system

In this section, we investigate the characteristics of the closed-loop system with the stabilizing controller given by (4.4).

First, the reference tracking characteristic is considered. If the parameterization of all stabilizing minimum-phase controllers for minimum-phase MIMO Biproper plants in (4.4) is employed, the transfer function from the reference input $r(s)$ to the output $y(s)$ of the control system in (4.1) is given by

$$y(s) = Q(s)r(s). \tag{4.13}$$

Therefore, for the output $y(s)$ to follow the step reference input

$$r(s) = \begin{bmatrix} \frac{1}{s} \\ \frac{1}{s} \end{bmatrix} \tag{4.14}$$

without steady-state error,

$$Q(0) = I \tag{4.15}$$

must be satisfied. If $Q(s)$ is chosen to satisfy (4.15) the output $y(s)$ follows the step reference input $r(s)$ without steady-state error.

Next, we consider the disturbance attenuation characteristic. The transfer function from the reference input $d(s)$ to the output $y(s)$ is given by

$$y(s) = (I - Q(s))r(s). \tag{4.16}$$

Therefore, to attenuate the step disturbance

$$d(s) = \begin{bmatrix} \frac{1}{s} \\ \frac{1}{s} \end{bmatrix} \tag{4.17}$$

completely,

$$I - Q(0) = 0 \tag{4.18}$$

must be satisfied. If $Q(s)$ is chosen to satisfy (4.18), the step disturbance $d(s)$ is attenuated effectively.

4.5 Design method of minimum-phase stabilizing controllers

In this section, we propose a design method of minimum-phase stabilizing controllers. From Theorem 4.3.1, to design a minimum-phase stabilizing controller $C(s)$, we need to settle $Q(s) \in RH_\infty^{m\times m}$ is any minimum-phase function to make $(I - Q(s))G(s) \in RH_\infty$. In addition, for the output $y(s)$ to follow the reference input $r(s)$ without steady-state error and the disturbance $d(s)$ attenuated effectively, $Q(s)$ needs to satisfy (4.15). A design method of $Q(s) \in RH_\infty^{m\times m}$ being any minimum-phase function to make $(I - Q(s))G(s) \in RH_\infty$ and stabilizing controller $C(s)$ is summarized as follows.

1. Let $G(s)$ be factorized as

$$G(s) = G_1(s)G_2(s), \tag{4.19}$$

   where $G_1(s) \in RH_\infty^{m\times m}$ is biproper of minimum-phase and contains all unstable poles and $G_2(s) \in RH_\infty^{m\times m}$ is biproper of minimum-phase and contains all stable poles.

2. Using $G_1(s)$, design $Q(s) \in RH_\infty^{m\times m}$ as

$$Q(s) = I - sG_1^{-1}(s)\frac{k}{(\tau s + 1)^\alpha}, \tag{4.20}$$

   where $\tau \in R$, α is an arbitrary positive integer to make $Q(s)$ proper, and k is a constant.

3. Using $Q(s)$ in (4.20), fix a stable stabilizing controller $C(s)$ in (4.4).

4.6 Numerical example

In this section, a numerical example is illustrated to show that a stabilizing minimum-phase controller written by the form of (4.4) can stabilize the minimum-phase plant.

Consider the problem to make the control system in (4.1) stable using stabilizing minimum-phase controller, where the plant $G(s)$ is written as

$$G(s) = \begin{bmatrix} \dfrac{s+2}{s-1} & \dfrac{5s+7}{s-1} \\ \dfrac{3s+5}{s-1} & \dfrac{4s+3}{s-1} \end{bmatrix}. \tag{4.21}$$

$G(s)$ is of minimum-phase and biproper. Using the design method described in Section 4.5, we obtain a minimum-phase stabilizing controller $C(s)$ to make the output $y(s)$ follow the step reference input $r(s)$ without steady state error and to attenuate the step disturbance $d(s)$ effectively. First $G(s)$ in (4.21) is factorized by (4.19), where

$$G_1(s) = \begin{bmatrix} \frac{s+2}{s-1} & \frac{5s+7}{s-1} \\ \frac{3s+5}{s-1} & \frac{4s+3}{s-1} \end{bmatrix} \tag{4.22}$$

and

$$G_2(s) = I. \tag{4.23}$$

Using $G(s)$ in (4.22), $Q(s) \in RH_\infty^{m\times m}$ is given by (4.20), where k, α, and τ are settled by

$$k = 1, \tag{4.24}$$

$$\alpha = 1 \tag{4.25}$$

and

$$\tau = 1. \tag{4.26}$$

Then, $Q(s)$ is obtained as

$$Q(s) = \begin{bmatrix} \dfrac{1.3636(s+1.21)(s^2+3.256s+4.792)}{(s+3)(s^2+3.182s+2.636)} & \dfrac{-0.45455s(s-1)(s+1.4)}{(s+3)(s^2+3.182s+2.636)} \\ \dfrac{-0.27273s(s-1)(s+1.667)}{(s+3)(s^2+3.182s+2.636)} & \dfrac{1.0909(s+2.557)(s^2+3.193s+2.836)}{(s+3)(s^2+3.182s+2.636)} \end{bmatrix}. \tag{4.27}$$

$Q(s)$ is of minimum-phase. From (4.21) and (4.27), $(1 - Q(s))G(s)$ is written as

$$(1 - Q(s))G(s) = \begin{bmatrix} \frac{s}{s+3} & 0 \\ 0 & \frac{s}{s+3} \end{bmatrix}. \tag{4.28}$$

$(1 - Q(s))G(s)$ is stable and proper. From (4.21) and (4.27), a stabilizing controller $C(s)$ written by the form of (4.4) to make the control system in (4.1) stable is written as

$$C(s) = \begin{bmatrix} \dfrac{1.3636(s+1.21)(s^2+3.256s+4.792)}{s(s^2+3.182s+2.636)} & \dfrac{-0.45455(s-1)(s+1.4)}{s^2+3.182s+2.636} \\ \dfrac{-0.27273(s-1)(s+1.667)}{s^2+3.182s+2.636} & \dfrac{1.0909(s+2.557)(s^2+3.193s+2.836)}{s(s^2+3.182s+2.636)} \end{bmatrix}. \tag{4.29}$$

$C(s)$ in (4.41) is of minimum-phase. Therefore if $C(s)$ in (4.41) makes $G(s)$ in (4.27) stable, then $C(s)$ in (4.41) is a stabilizing minimum-phase controller for the minimum-phase plant $G(s)$ in (4.27).

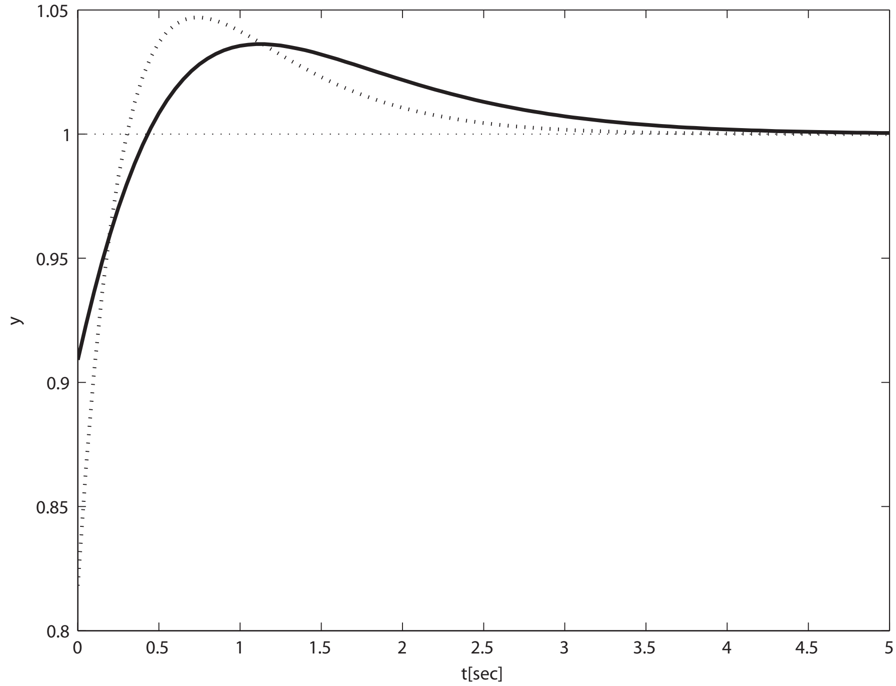

Fig. I: Response of the output $y(t)$ of the control system in (4.1) for the step reference input $r(t)$

Using the stabilizing minimum-phase controller $C(s)$ in (4.41), the response of the output $y(t)$ of the control system in (4.1) for the step reference input $r(t)$ is shown in Figure I. Figure I shows that the control system in (4.1) is stabilized by using a stabilizing minimum-phase controller $C(s)$ in (4.41) and that the output $y(s)$ follows the step reference input $r(s)$ without steady-state error.

In contrast, when the step disturbance $d(s)$ is exerted, the response of the output $y(s)$ of the closed-loop system (4.1) is depicted in Figure II. Figure II verifies that the disturbance $d(s)$ is effectively attenuated.

The presented example shows that the proposed method can design a minimum-phase stabilizing controller $C(s)$ based on reference tracking and the disturbance attenuation characteristics.

In this way, we find that if the stabilizing controller $C(s)$ is written by the form of (4.4), the minimum-phase plant is stabilized.

4.7 The Parameterization of All Stabilizing Minimum-Phase Controllers for Minimum-Phase Strictly Proper Plants

In this section, we clarify the parameterization of all stabilizing minimum-phase controllers $C(s)$ for minimum-phase strictly proper plants $G(s)$.

This parameterization is summarized in the following theorem.

Theorem 4.7.1 *$G(s)$ is assumed to be of minimum-phase and strictly proper. There exists $K(s)$ that satisfies the following expressions: (1)$G(s)+K(s)$ is of minimum phase. (2)$K(s)\in RH_\infty^{m\times m}$ is biproper. Using above mentioned $K(s)$, the parametrization of all proper stabilizing controllers $C(s)$ for $G(s)$ is given by*

$$C(s)=\left((I+\bar{C}(s))K(s)\right)^{-1}\bar{C}(s)\quad\left(\det\{\lim_{\omega\to\infty}\left((I+\bar{C}(j\omega))K(j\omega)\right)\}\neq 0\right), \tag{4.30}$$

where $\bar{C}$ is denoted by

$$\bar{C}(s)=\left((I-\bar{Q}(s))(G(s)+K(s))\right)^{-1}\bar{Q}(s) \tag{4.31}$$
$$\left(\det\{\lim_{\omega\to\infty}\left((I-\bar{Q}(j\omega))(G(j\omega)+K(j\omega))\right)\}\neq 0\right)$$

and $\bar{Q}(s)\in RH_\infty^{m\times m}$ is any minimum-phase function to make $(I-\bar{Q}(s))(G(s)+K(s))\in RH_\infty$.

The proof of Theorem 4.7.1 requires the following theorems.

Theorem 4.7.2 *If $G(s)$ is of minimum-phase and strictly proper, there exists $K(s)$ that satisfies the following expressions: (1)$G(s)+K(s)$ is of minimum-phase and biproper. (2)$K(s)\in RH_\infty^{m\times m}$ is biproper.*

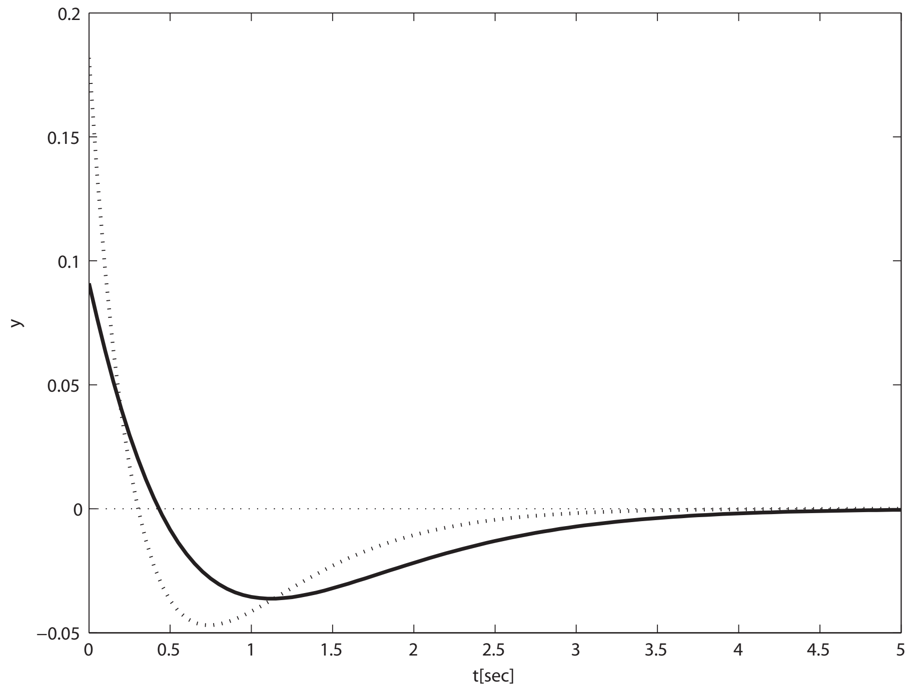

Fig. II: Response of the output $y(t)$ of the control system in (4.1) for the step disturbance $d(t)$

(Proof)

$G(s) + K(s)$ is written as

$$G(s) + K(s) = (N(s) + K(s)D(s))\, D(s)^{-1}, \tag{4.32}$$

where $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ are right coprime factors of $G(s)$ over RH_∞ written by

$$G(s) = N(s)D(s)^{-1}. \tag{4.33}$$

The existence condition of $K(s) \in RH_\infty$ to satisfy $N(s) + K(s)D(s) \in \mathcal{U}$ is equivalent to that of $U(s) \in \mathcal{U}$ and $K(s) \in RH_\infty$ satisfying

$$U(s) = N(s) + K(s)D(s) \in \mathcal{U}, \tag{4.34}$$

where $\mathcal{U}$ is the set of unimodular matrices on RH_∞, thus $U(s) \in \mathcal{U}$ means $U(s) \in RH_\infty$ and $U^{-1}(s) \in RH_\infty$. Applying Theorem 4.4.1 in [7] for (4.34), the existence condition of $U(s)$ and $K(s)$ is reduced to that of $u(s) \in \mathcal{U}$ and $k(s) \in RH_\infty$ satisfying

$$u(s) = n(s) + k(s)d(s) \in \mathcal{U}, \tag{4.35}$$

where $n(s) = \det(N(s)) \in RH_\infty$, $d(s) \in RH_\infty$ denotes the smallest invariant factor of $D(s)$ and $k(s)$ satisfies

$$\det\{N(s) + K(s)D(s)\} = n(s) + k(s)d(s). \tag{4.36}$$

The existence condition of $u(s)$ and $k(s)$ is equivalent to the interpolation problem written by

$$\frac{d^j}{ds^j}u(s_i) = \frac{d^j}{ds^j}n(s_i) \quad (j = 0, \ldots, m_i - 1; i = 1, \ldots, l), \tag{4.37}$$

where $s_1, \ldots, s_l$ are distinct zeros of $d(s)$ on the positive real axis, $m_1, \ldots, m_l$ are the corresponding multiplicities and l denotes the number of distinct zeros of $d(s)$ on the positive real axis. Since $G(s)$ is of minimum-phase, $n(s)$ is also of minimum-phase. This implies that all of $n(s_i)$ are the same sign. From Theorem 2.3.1 in [7], there exists $u(s) \in \mathcal{U}$ and $k(s) \in RH_\infty$ satisfying (4.37). This implies that there exists $U(s) \in \mathcal{U}$ and $K(s) \in RH_\infty$ satisfying (4.34).

The remaining problem is whether or not, $K(s)$ is biproper. Next, it is shown that if $U(s) \in \mathcal{U}$ exists such that (4.34) holds true, then $K(s)$ is biproper. From (4.34), $K(s)$ is written by

$$K(s) = (U(s) - N(s))D(s)^{-1}. \tag{4.38}$$

The assumption that $U(s)$ holds (3.6) implies that $K(s)$ written by (3.8) is stable. Since both $U(s)$ and $D(s)$ are biproper and $N(s)$ is strictly proper, $K(s)$ denoted by (3.8) is biproper.

This completes the proof of this theorem.

Theorem 4.7.3 *If $\bar{G}(s) = G(s) + K(s)$ is of minimum-phase and biproper, then the parametrization of all proper minimum-phase stabilizing controllers $\bar{C}(s)$ for $\bar{G}$ is written by*

$$\bar{C}(s) = \left((I - \bar{Q}(s))\bar{G}(s)\right)^{-1} \bar{Q}(s) \quad \left(\det\{\lim_{\omega\to\infty} ((I - \bar{Q}(j\omega))\bar{G}(j\omega))\} \neq 0\right), \tag{4.39}$$

where $\bar{Q}(s) \in RH_\infty^{m\times m}$ is any minimum-phase function to make $(I - \bar{Q}(s))\bar{G}(s) \in RH_\infty$.

(Proof)

Since $\bar{G}(s) = G(s) + K(s)$ is assumed to be minimum-phase and biproper, from Theorem 4.3.1, it is obvious that the parametrization of all stabilizing controllers for $\bar{G}(s) = G(s) + K(s)$ is given by (4.39).

This completes the proof of this theorem.

Theorem 4.7.4 *It is assumed that $K(s) \in RH_\infty$ is biproper and $G(s)$ is strictly proper. If the minimum-phase controller $C(s)$ stabilizes $G(s)$, then $\bar{C}(s)$ is written by*

$$\bar{C}(s) = (I - C(s)K(s))^{-1} C(s) \quad \left(\det\{\lim_{\omega\to\infty} (I - C(j\omega)K(j\omega))\} \neq 0\right) \tag{4.40}$$

stabilizes $\bar{G}(s) = G(s) + K(s)$. In addition, the reverse is also true. That is, if the minimum-phase controller $\bar{C}(s)$ stabilizes $\bar{G}(s) = G(s) + K(s)$, then the the minimum-phase controller $C(s)$ is written by

$$C(s) = \left(I + \bar{C}(s)K(s)\right)^{-1} \bar{C}(s) \quad \left(\det\{\lim_{\omega\to\infty} \left(I + \bar{C}(j\omega)K(j\omega)\right)\} \neq 0\right) \tag{4.41}$$

stabilizes $G(s)$.

(Proof)

First, we will show that if the minimum-phase controller $C(s)$ stabilizes $G(s)$, then the minimum-phase controller $\bar{C}(s)$ written by (4.40) stabilizes $\bar{G}(s) = G(s) + K(s)$. $K(s)$ is assumed to be biproper and $C(s)$ is assumed to be of minimum-phase. In (4.40) if the $(I - C(s)K(s))^{-1}$ has unstable zeros, the unstable zeros are the unstable poles of $C(s)$. Therefore, the $\bar{C}(s)$ has no unstable zeros, that is, $\bar{C}(s)$ is of minimum-phase. Then from (4.40) and simple manipulation, $(I + \bar{G}(s)\bar{C}(s))^{-1}$, $(I + \bar{G}(s)\bar{C}(s))^{-1}\bar{G}(s)$, $\bar{C}(s)(I + \bar{G}(s)\bar{C}(s))^{-1}$ and $(I + \bar{G}(s)\bar{C}(s))^{-1}\bar{G}(s)\bar{C}(s)$ are rewritten by

$$(I + \bar{G}(s)\bar{C}(s))^{-1} = (I + (G(s) + K(s))C(s))^{-1}(I - K(s)C(s)), \tag{4.42}$$

$$(I + \bar{G}(s)\bar{C}(s))^{-1}\bar{G}(s) = (I + (G(s) + K(s))C(s))^{-1}(I - K(s)C(s))(G(s) + K(s)), \tag{4.43}$$

$$\bar{C}(s)(I + \bar{G}(s)\bar{C}(s))^{-1} = C(s)(I + G(s)C(s))^{-1}, \tag{4.44}$$

and

$$(I + \bar{G}(s)\bar{C}(s))^{-1}\bar{G}(s)\bar{C}(s) = (I + G(s)C(s))^{1}(G(s) + K(s))C(s). \tag{4.45}$$

From the assumption that $C(s)$ stabilizes $G(s)$, $(I+G(s)C(s))^{-1}$, $C(s)(I+G(s)C(s))^{-1}$, $(I+G(s)C(s))^{-1}G(s)$ and $(I + G(s)C(s))^{-1}G(s)C(s)$ are all include in RH_∞. Therefore, all of transfer functions in (4.42), (4.43), (4.44) and (4.45) are include in RH_∞.

Next, we will show that if the minimum-phase controller $\bar{C}(s)$ stabilizes $\bar{G}(s) = G(s) + K(s)$, then the minimum-phase controller $C(s)$ written by (4.41) stabilizes $G(s)$. In (4.41) if the $(I+\bar{C}(s)K(s))^{-1}$ has unstable zeros, the unstable zeros are the unstable poles of $C(s)$. Therefore, the $C(s)$ has no unstable zeros, that is, $C(s)$ is of minimum-phase. Then from (4.41) and simple manipulation, $(I + G(s)C(s))^{-1}$, $C(s)(I + G(s)C(s))^{-1}$, $(I + G(s)C(s))^{-1}G(s)$ and $(I + G(s)C(s))^{-1}G(s)C(s)$ are rewritten by

$$(I + G(s)C(s))^{-1} = (I + (\bar{G}(s) - K(s))\bar{C}(s))^{-1}(I + \bar{C}(s)K(s)), \tag{4.46}$$

$$C(s)(I+G(s)C(s))^{-1} = (I+(\bar{G}(s)-K(s))\bar{C}(s))^{-1}(I+K(s)\bar{C}(s))(\bar{G}(s)-K(s)), \tag{4.47}$$

$$(I+G(s)C(s))^{-1}G(s) = \bar{C}(s)(I+\bar{G}(s)\bar{C}(s)), \tag{4.48}$$

and

$$(I+G(s)C(s))^{-1}G(s)C(s) = (I+\bar{G}(s)\bar{C}(s))^{-1}((\bar{G}(s)-K(s))\bar{C}(s)). \tag{4.49}$$

From the assumption that $\bar{C}(s)$ stabilizes $\bar{G}(s) = G(s) + K(s)$, $(I + G(s)C(s))^{-1}$, $C(s)(I + G(s)C(s))^{-1}$, $(I+G(s)C(s))^{-1}G(s)$ and $(I+G(s)C(s))^{-1}G(s)C(s)$ are all include in RH_∞. Therefore, all of transfer functions in (4.46), (4.47), (4.48) and (4.49) are include in RH_∞.

We have thus proved Theorem 3.3.4.

Theorem 3.3.1 is proved using the above mentioned theorems.

From Theorem 4.7.2, there exists biproper $K(s) \in RH_\infty$ to make $\bar{G}(s) = G(s) + K(s)$ of minimum phase. From Theorem 4.7.4, the parametrization of all internally stabilizing controllers $C(s)$ for $G(s)$ is same to that of all internally stabilizing controllers $\bar{C}(s)$ for $\bar{G}(s) = G(s)+K(s)$. The parametrization of all internally stabilizing controllers $\bar{C}(s)$ for $\bar{G}(s) = G(s) + K(s)$ is given by (4.39), where $\bar{Q}(s) \in RH_\infty$ is any minimum-phase function to make $(I - \bar{Q}(s))\bar{G}(s) \in RH_\infty$. Equation (4.39) corresponds to (4.31). From Theorem 4.7.4, using $\bar{C}(s)$, $C(s)$ is written by (4.41). Equation (4.41) corresponds to (4.30). This completes the proof of Theorem 4.7.1.

4.8 Conclusion

In this chapter, we clarified the parameterization of all stabilizing minimum-phase controllers for minimum-phase MIMO plants. That is, we showed that if the stabilizing controller $C(s)$ is written by the form of (4.4), the minimum-phase plant is stabilized. In addition, we showed a numerical example to illustrate that a stabilizing minimum-phase controller written by the form of (4.4) can stabilize the minimum-phase plant. We will present the parameterization of all stabilizing minimum-phase controllers for minimum-phase MIMO strictly proper plants.

Chapter 5

Conclusion

In this thesis, we propose the parameterization of all stabilizing minimum-phase controllers for minimum-phase plants.

In chapter 2, we clarified the parameterization of all stabilizing minimum-phase controllers for minimum-phase biproper plants. That is, we showed that if the stabilizing controller $C(s)$ is written by the form of (**??**), the minimum-phase plant is stabilized. We also present a design method of the minimum-phase stabilizing controllers for the minimum-phase biproper plants that satisifies the robustness of the control system according to the internal model principle. In addition, we show a numerical example to illustrate the features of the proposed design method. In order to check the robustness of this example, we considered this situation that the minimum-phase controller stabilizes the perturbed plant. And we compare the proposed design method with that of [25]. By using the same $Q(s)$, the controller that we obtained by the proposed method is of minimum- phase. In contrast, by using the previous method, the controllers that we obtain are not necessarily of minimum-phase.

In chapter 3, we propose the parameterization of all minimum-phase stabilizing controllers for minimum-phase stabilizing strictly proper single-input single-output plants. Analysis of the internal stability and control characteristics of closed-loop system are provided. We also present the control system characteristics with the parameterization of all stabilizing minimum-phase controllers for minimum-phase strictly proper plants. The reference tracking characteristic and the disturbance attenuation characteristic is considered. In additon, a numerical example is illustrated for the effectiveness of the proposed method and the robustness of this example is checked by using a perturbed plant.

In chapter 4, we propose the parameterization of all minimum-phase stabilizing controllers for minimum-phase stabilizing multiple-input multiple-output plants. In this chapter, we extend the conclusions of the previous two chapters to multiple-input multiple-output systems. The control system characteristics with the parameterization of all stabilizing minimum-phase controllers for minimum-phase biproper multiple-input multiple-output plants is shown. In addition, we propose a design method of stabilizing minimum-phase controllers based on the parameterization of all stabilizing minimum-phase controllers for minimum-phase plants satisifying the robustness. We also show a numerical example to illustrate the features of the proposed design method. At last, we clarify the parameterization of all stabilizing minimum-phase controllers for minimum-phase strictly proper multiple-input multiple-output plants. In this way we obtain the parameterization of all stabilizing minimum-phase controllers for minimum-phase multiple-input multiple-output plants.

In future work, A design method of a stabilizing minimum-phase controllers for unknown minimum-phase plants will be considered.

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Publication List

Chapter 2

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Chapter 3

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