群馬大学博士論文

# Study on semistrongly stabilizing controllers

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# Contents

# Chapter 1

# Introduction

## 1.1 Controller design

One of the important things in control is the controller design. The main role of controller is to stabilize the control system and to make the control characteristic close to the ideal one. If there is the enough know-how to design such controller, it is easy to design the controller with those roles. Otherwise, in order to make the controller have those roles, the tuning of controller is needed. However, when the designed controller is tuned, the stability of control system is generally not guaranteed. Therefore, there are cases when the controller needs to design from the beginning.

Based on this problem, a method to guarantee the stability of control system firstly is considered. This method is called two-steps method. The processure is to settle the set of all stabilizing controllers for the plant firstly, and select one in that set secondly. Because the control system stability is guaranteed, we need no consideration to that stability on the controller design. In the research on control theory, one way to design stabilizing controller by two-steps method is to parameterize the set of all stabilizing controllers, so-called parameterization.

### 1.1.1 Parameterization

The first parameterization on the stabilizing controller is proposed by Youla et al. [1]. In their parameterization, all stabilizing controller to guarantee the stability of feedback control system in Fig. 1.1 is clarified. Here, $r$ is the reference input, $C(s)$ is the controller, $G(s)$ is the plant,

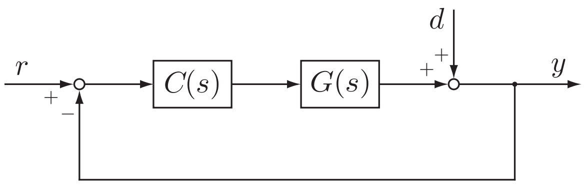


Fig. 1.1: Feedback control system

$d$ is the disturbance and $y$ is the output. The design processure using the parameterization of all stabilizing controllers is as follows.

1. Coprime factors of the plant $G(s)$ is obtained by plant modeling. Using these factors, the set of all stablizing controller to guarantee the stability of control system is obtained.

2. By selecting the free parameter to realize the required control performance, the stabilizing controller is given.

Using the parameterization of all stabilizing controllers, the control system is always stable. Therefore, there is no necessity to consider the control system stability on the controller design. In addition, because the control performance can be decided by one free parameter, it is easy to tune. Youla et al. also present such parameterization for multiple-input/multiple-output plants [2]. Their research [1, 2] makes a substantial contribution and is refered by many reports on control theory. In addition, many parameterizations have been published, for example, PID [3], internal model controllers [4, 5, 6] and disturbance observers [7]. From this, the importance to clarify parameterizations is seen.

However, most of the researches on the parameterization do not consider the stability of resulting stabilizing controller. In practical control, there are cases when the control system needs to be stabilized by stable controllers. The control system stabilization by stable controllers is called strong stabilization. The importance of strong stabilization is shown by the relationship between the controller stability and sensitivity and the safety control.

### 1.1.2 Controller stability and sensitivity

The relation between the controller stability and the sensitivity of control system is clarified by Shaw [8]. His research points out that the control system stabilized by unstable controller is high sensitive for variations in plant parameters. This causes the unstability and poor performance of control system. Therefore, not only the stability of control system but also that of stabilizing controller must be paid attention. In addition, Shaw [8] also shows the existence of plant, which is stabilizable by only unstable stabilizing controller. That is, even if the plant is stabilizable by stabilizing controller, this plant is not always stabilizable by stable one. Thus it needs to see whether the plant is stabilizable by stable controller or not, before the controller design.

### 1.1.3 Reliable control

One reason that the controller stability is important is the safety control to suppose the trouble in control system. This safety control is called reliable control [9, 10]. As the system is required the advanced processing, then it becomes complicated and enormous. From this, control devices such as sensors, controllers and actuators used for recent plants are increasing. Under this background, the reliable control is remarked because it is difficult to guarantee the best condition of control devices. In this subsection, as the important category in reliable control, the simultaneous stabilization and the passive redundancy are shown.

First, the simultaneous stabilization is shown. The simultaneous stabilization is a control method to stabilize the plural plant by one stabilizing controller. In the reliable control, the change of control system by the trouble of plant, sensor and so on is supposed beforehand, and the controller is also designed beforehand to stabilize control system before and after troubles [11]. Vidyasagar and Viswanadham [11] show that the simultaneous stabilization can be came down to the strong stabilization problem. In particular, the simultaneous stabilization for plants $G_1(s), G_2(s), \ldots, G_n(s)$ can be came down the problem to strongly stabilize $n-1$ transfer functions [12].

Next, the passive redundancy is shown. The passive redundancy is a control method to stabilize one plant by the plural control device such as controllers and actuators. This control method supposes beforehand the change of control system by the trouble of control device, and inserts the reserve into the control system. The interference by reserve control devices may cause the unstability of control system. Therefore, some researches [10, 13, 14] examine redundant stabilizing controllers. Minto and Ravi show that the passive redundancy problem can be also came down to the strong stabilization problem [13]. That is, the passive redundancy problem for stabilizing controllers $C_1(s), C_2(s), \ldots, C_n(s)$ can be came down the problem to strongly stabilize $n-1$ transfer functions.

In this way, the strong stabilization concerns the low sensitivity of control system and the reliable control, that is, concerns the practical control. In this thesis, the strong stabilization is thus focused.

## 1.2 Strong stabilization

In this section, how the strong stabilization, which is the deep involvement with semistrong stabilization, has been researched is introduced.

Because the existence of stabilizable plants by only unstable controllers is shown by Shaw [8], it is necessary to clarify the way whether the plant is strongly stabilizable or not. For this problem, Youla et al. [15] shows the necessary and sufficient condition that the plant is strongly stabilizable in the following feedback control system.

1. The plant $G(s)$, the controller $C(s)$ and the feedback sensor $F(s)$ are finite-dimensional linear time variant dynamical systems.

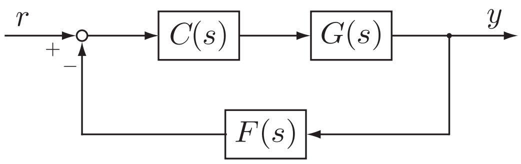


Fig. 1.2: Feedback control system with feedback sensor

2. Uncertain modes of the plant $G(s)$ is asymptotic stable.

3. The controller $C(s)$ and the feedback sensor $F(s)$ are completely observable and controllable.

4. The closed loop is dynamical.

The necessary and sufficient condition published by Youla et al. is called p.i.p.(parity interlacing property). This implies that the plant is strongly stabilizable if and only if numbers of real unstable poles of the plant between each real zeros of the plant in closed right half plane on complex plane are all even. From this result, the common characteristic of all strongly stabilizable plants has been clarified. In addition, Youla et al. [15] also present the design method of strongly stabilizing controller using the Nevanlinna-Pick interpolation. In this way, their research enable to see whether the plant is strongly stabilizable or not, and design the strongly stabilizing controller.

Smith and Sondergeld [16] points out that there are cases when the resulting strongly stabilizing controller designed by above method [15] is very high-order and irrational function. Such controllers are inappropriate for the practical control. Therefore, some researchs [17, 18, 19] solve this problem. In these researches, some methods to design low-order and rational strongly stabilizing controller are presented. The design method for multiple-input/multiple-output plants is also presented [20]. In addition, using the strongly stabilizing controller, several controller designs to make control system be low sensitive are considered [19, 21]. Moreover, the strong stabilization is expanded for several control theory. Halevi [22] examines for the stable LQG(Linear-Quadratic-Gaussian) controller design. LQG is a control method to combine the optimal control and Kalman filter, and to estimate the optimal state of plant. Banksa et al. [23] examine for the fuzzy control, which applies the fuzziness like humans and is used for the control of consumer electronics. Gumussoy and Ozbay [25] examine the sensitivity minimization for time-delay systems such as the remote control via network. Wakaiki et al. examine the sensitivity reduction using the matrix Nevanlinna-Pick interpolation for infinite dimensional systems such as elastic vibration and heat conduction [26]. They also examine this sensitivity reduction using the tangential Nevanlinna-Pick interpolation [27]. Furthermore, in order to parameterize the set of all strongly stabilizing controllers, the parameterization of all strongly stabilizable plants is proposed [28]. Using this result, the parameterization of all strongly stabilizing controllers is also presented [29].

In this way, some researches on strongly stabilizing controller are published.

## 1.3 Semistrong stabilization

In this section, why the semistrong stabilization has been examined is shown. With strongly stabilizing controllers, when there is an uncertainty in the plant or a step disturbance, the output of the control system cannot follow the step reference input without steady state error. The reason is that strongly stabilizing controllers cannot have an origin poles. If the control requires high tracking performance, stabilizing controllers require an integrator. Therefore, it is necessary to examine controller designs that have an origin pole and other poles in the open left-half plane. Such controllers are called semistrongly stabilizing controllers [28].

For this research, we need to examine the parameterization of all semistrongly stabilizing controllers in order to present a controller design guaranteeing the stability of control system. However, because the existence of plants that cannot be stabilized by strongly stabilizing controllers is clarified [15], the existence of plants that cannot be stabilized by semistrongly stabilizing controllers is expected. Therefore, we need to examine the parameteization of all semistrongly stabilizable plants before the examination of parameterization of all semistrongly stabilizing controllers.

## 1.4 The purpose and contents of this study

In this thesis, parameterizations on the semistrong stabilization are proposed.

In Chapter 2, we propose the parameterization of all semistrongly stabilizable plants. A numerical example is presented to show that the plant in the proposed parameterization is surely semistrongly stabilizable.

In Chapter 3, we propose the parameterization of all semistrongly stabilizing controllers for semistrongly stabilizable plants in Chapter 2. Control characteristics and a design method of the semistrongly stabilizing controller are also presented. A numerical example is illustrated to show that the controller designed by the presented design method surely works as semistrongly stabilizing controller. In this example, the angular velocity control of the two-inertia system is adopted in order to show the efficiency for the practical control.

In Chapter 4, we propose the parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers for semistrongly stabilizable plants in Chapter 2. With the parameterization in Chapter 3, we cannot specify the input–output characteristic and the feedback characteristic, for example, a disturbance attenuation characteristic and robust stability, separately. One way to specify those characteristics separately is to use a two-degrees-of-freedom control system. Therefore, the purpose of this chapter is to propose the parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers. Control characteristics and a design method of the two-degrees-of-freedom semistrongly stabilizing controller are also presented. A numerical example is illustrated to show the effectiveness for the proposed parameterization in Chapter 3 by comparison for responses of the numerical example in Chapter 3.

Chapter 5 summarizes the result of the present study by the conclusion.

## Notations

| | |
|---|---|
| $R$ | The set of real numbers. |
| $R(s)$ | The set of real rational function with $s$. |
| $RH_\infty$ | The set of stable proper real rational functions. |
| $\mathcal{U}$ | The set of unimodular functions on $RH_\infty$. That is, $U(s) \in \mathcal{U}$ implies both $U(s) \in RH_\infty$ and $U^{-1}(s) \in RH_\infty$. |
| $\\|\{\cdot\}\\|_\infty$ | The norm of $\{\cdot\}$. |

# Chapter 2

# The parameterization of all semistrongly stabilizable plants

## 2.1 Introduction

In the parameterization problem, all stabilizing controllers for a plant [1, 2, 4, 7, 30, 31, 12, 32, 33, 34, 35] and all plants that can be stabilized [3] are sought. Because this parameterization can successfully search for all proper stabilizing controllers, it is used as a tool for many control problems.

In practical control problems, both the stability of the closed-loop systems and that of the stabilizing controllers are important. In certain cases [8], the instability of stabilizing controllers causes poor overall system sensitivity to variations in plant parameters. On the other hand, from [15], even if a plant is stabilizable, the plant is not necessarily strongly stabilizable. In addition, the achievable control characteristic is restricted in comparison with the case using unstable controllers. It is thus desirable to choose either a stable controller or an unstable one by the required control specification. Since nonstrongly stabilizable plants exist, two necessary and sufficient conditions that a plant is strongly stabilizable have been clarified. One was clarified by Youla et al. and is called the parity interlacing property, which is a condition on the placement of poles and zeros of strongly stabilizable plants [15, 12]. They also proposed a method to find strongly stabilizing controllers using Nevanlinna–Pick interpolation [15, 12]. This result was developed further in several papers about the design method for strongly stabilizing controllers [16, 17, 19, 25]. The other condition was clarified by Hoshikawa et al. and is the parameterization of all strongly stabilizable plants, which shows that strongly stabilizable plants have a common feedback structure [28]. They also proposed the parameterization of all strongly stabilizing controllers, thus enabling the systematic design of strongly stabilizing controllers. The strong stabilization problem has thus been studied extensively.

With strongly stabilizing controllers, when there is an uncertainty in the plant or a step disturbance, the output of the control system cannot follow the step reference input without steady state error. The reason is that strongly stabilizing controllers cannot have a pole at the origin. If the control requires high tracking performance, stabilizing controllers require an integrator. Therefore, it is necessary to examine controller designs that have a pole at the origin and other poles in the open left-half plane. Such controllers is called semistrongly stabilizing controllers [28]. Because plants that are unstabilizable by strongly stabilizing controllers exist [15], it is expected that plants that cannot be stabilized by a semistrongly stabilizing controller also exist. Therefore, the parameteization of all semistrongly stabilizable plants is needed to examine before the examination of parameterization of all semistrongly stabilizing controllers.

In this chapter, we examine the parameterization of all semistrongly stabilizable plants.

## 2.2 Problem formulation

Consider the control system:

$$\begin{cases} y(s) &= G(s)u(s) + d(s) \\ u(s) &= C(s)\left(r(s) - y(s)\right) \end{cases}, \tag{2.1}$$

where $G(s) \in R(s)$ is the plant, $C(s) \in R(s)$ is the controller, $y(s)$ is the output, $u(s)$ is the control input, $d(s)$ is the disturbance and $r(s)$ is the reference input.

The semistrong stabilization is a control problem to stabilize control system by the strongly stabilizing controller with an origin pole. That is, the concept of semistrongly stabilizing controllers is defined as follows.

**Definition 1** *(Semistrongly stabilizing controllers)[28]*
*We call the controller $C(s)$ in (2.1) a "semistrongly stabilizing controller" if the stabilizing controller has only one pole at the origin and other poles in the open left-half plane. That is, if $C(s)$ in (2.1) is written by:*

$$C(s) = \frac{s+\alpha}{s} Q_1(s), \tag{2.2}$$

*then we call $C(s)$ in (2.1) a semistrongly stabilizing controller, where $\alpha \in R$ is any positive real number and $Q_1(s) \in RH_\infty$ is any function satisfying $Q_1(0) \neq 0$.*

From Definition 1, we see that the semistrongly stabilizing controller has stable poles and only one origin pole. The aim of the semistrong stabilization is to prevent that the excessive high-sensitivity of the control system by the unstable poles of stabilizing controllers, and to make the output of the control system follow the step reference input without steady state error in the presence of an uncertainty in the plant or a step disturbance. In addition, semistrongly stabilizable plants is defined as follows.

**Definition 2** *(Semistrongly stabilizable plant)[28]*
*We call $G(s)$ in (2.1) a "semistrongly stabilizable plant" if $G(s)$ in (2.1) can be stabilized by a semistrongly stabilizing controller $C(s)$ in (2.2).*

Because semistrongly stabilizing controller must have no unstable poles except for one origin pole, the achievable control performance is limited compared with the stabilizing controller allowed unstable poles. Therefore, it can be estimated even if a plant is stabilizable, that plant is not always semistrongly stabilizable. From this reason, the set of all semistrongly stabilizable plants have to be clarified as the first step of the study on the semistrong stabilization. One of the ways to obtain the set of all semistrongly stabilizable plants is to express all condition of this set by parameters, so colled parameterization [1, 2, 12].

The purpose of this chapter is to clarify the parameterization of all semistrongly stabilizable plants.

## 2.3 The parameterization of all semistrongly stabilizable plants

In this section, the parameterization of all semistrongly stabilizable plants $G(s)$ defined in Definition 2 is clarified.

The parameterization of all semistrongly stabilizable plants is summarized in the following theorem.

**Theorem 1** *The plant $G(s)$ is semistrongly stabilizable if and only if the plant $G(s)$ takes the form of:*

$$G(s) = \frac{sQ_2(s) + \beta}{(s+\alpha)\left(1 + Q_3(s) - Q_1(s)Q_2(s)\right)}, \tag{2.3}$$

*where $\beta \in R$ is written by:*

$$\beta = \frac{\alpha}{Q_1(0)}, \tag{2.4}$$

*$Q_3(s) \in RH_\infty$ satisfies:*

$$Q_3(s) = \frac{\alpha - \beta Q_1(s)}{s}, \tag{2.5}$$

*and $Q_1(s) \in RH_\infty$ and $Q_2(s) \in RH_\infty$ are any functions satisfying $Q_1(0) \neq 0$.*

Proof of Theorem 1 requires following lemma.

**Lemma 1** *Suppose that $A(s) \in RH_\infty^{n\times m}$, $B(s) \in RH_\infty^{q\times m}$, $C(s) \in RH_\infty^{p\times m}$, $\mathrm{rank}\begin{bmatrix} A^T(s) & B^T(s)\end{bmatrix}^T = r$. The equation written by:*

$$X(s)A(s) + Y(s)B(s) = C(s) \tag{2.6}$$

*has a solution $X(s)$ and $Y(s)$ if and only if there exists $U(s) \in \mathcal{U}$ to satisfy:*

$$\begin{bmatrix} A(s) \\ B(s) \\ C(s) \end{bmatrix} = U(s)\begin{bmatrix} A(s) \\ B(s) \\ 0 \end{bmatrix}. \tag{2.7}$$

*When a pair of $X_0(s)$ and $Y_0(s)$ is a solution to (2.6), all solutions are given by:*

$$\begin{bmatrix} X(s) & Y(s) \end{bmatrix} = \begin{bmatrix} X_0(s) & Y_0(s) \end{bmatrix} + Q(s)\begin{bmatrix} W_1(s) & W_2(s) \end{bmatrix}, \tag{2.8}$$

*where $W_1(s) \in RH_\infty^{p\times n}$ and $W_2(s) \in RH_\infty^{p\times q}$ are functions satisfying:*

$$W_1(s)A(s) + W_2(s)B(s) = 0 \tag{2.9}$$

*and*

$$\mathrm{rank}\begin{bmatrix} W_1(s) & W_2(s) \end{bmatrix} = n + q - r \tag{2.10}$$

*and $Q(s) \in RH_\infty^{p\times(n+q-r)}$ is any function [12].*

Using above-mentioned Lemma 1, Theorem 1 is proved.
*(Proof)* First, the necessity is shown. That is, we show that if $C(s)$ in (2.2) makes the control system in (2.1) stable, then $G(s)$ takes the form of (2.3). Coprime factorizations of $G(s)$ and $C(s)$ in (2.2) are denoted by:

$$G(s) = \frac{N(s)}{D(s)} \tag{2.11}$$

and

$$C(s) = \frac{N_c(s)}{D_c(s)}, \tag{2.12}$$

respectively. Here, $N(s) \in RH_\infty$, $D(s) \in RH_\infty$,

$$N_c(s) = Q_1(s) \in RH_\infty \tag{2.13}$$

and

$$D_c(s) = \frac{s}{s+\alpha} \in RH_\infty, \tag{2.14}$$

respectively, and $\alpha \in R$ and $Q_1(s)$ are in (2.2). From the assumption that $C(s)$ in (2.2) makes the control system in (2.1) stable, $N(s)N_c(s)+D(s)D_c(s) \in \mathcal{U}$. That is, from (2.13) and (2.14),

$$\begin{aligned} N(s)N_c(s) + D(s)D_c(s) &= N(s)Q_1(s) + D(s)\frac{s}{s+\alpha} \\ &= 1. \end{aligned} \tag{2.15}$$

From Lemma 1, all solutions $N(s)$ and $D(s)$ satisfying (2.15) are written by:

$$N(s) = \frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s) \tag{2.16}$$

and

$$D(s) = 1 + Q_3(s) - Q_1(s)Q_2(s). \tag{2.17}$$

Because:

$$\frac{\beta}{s+\alpha}Q_1(s) + (1+Q_3(s))\frac{s}{s+\alpha} = 1 \tag{2.18}$$

and

$$\frac{s}{s+\alpha}Q_1(s) - Q_1(s)\frac{s}{s+\alpha} = 0, \tag{2.19}$$

where $\beta$ and $Q_3(s)$ are written by (2.4) and (2.5), respectively, and $Q_2(s) \in RH_\infty$ is any function.

Substituting (2.16) and (2.17) for (2.11), we have:

$$G(s) = \frac{sQ_2(s) + \beta}{(s+\alpha)\left(1 + Q_3(s) - Q_1(s)Q_2(s)\right)}, \tag{2.20}$$

Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, if $G(s)$ in (2.1) takes the form of (2.3), then there exists a semistrongly stabilizing controller to make the control system in (2.1) stable. A controller is set as:

$$C(s) = \frac{s+\alpha}{s}Q_1(s). \tag{2.21}$$

From simple manipulation and (2.21), transfer functions $C(s)G(s)/(1+C(s)G(s))$, $C(s)/(1+C(s)G(s))$, $G(s)/(1+C(s)G(s))$ and $1/(1+C(s)G(s))$ are rewritten by:

$$\frac{C(s)G(s)}{1+C(s)G(s)} = \frac{sQ_2(s)+\beta}{s+\alpha}Q_1(s), \tag{2.22}$$

$$\frac{C(s)}{1+C(s)G(s)} = \left(1+Q_3(s) - Q_1(s)Q_2(s)\right)Q_1(s), \tag{2.23}$$

$$\frac{G(s)}{1+C(s)G(s)} = \frac{s\left(sQ_2(s)+\beta\right)}{(s+\alpha)^2} \tag{2.24}$$

and

$$\frac{1}{1+C(s)G(s)} = \frac{s\left(1+Q_3(s)-Q_1(s)Q_2(s)\right)}{s+\alpha}, \tag{2.25}$$

respectively. Because $Q_1(s) \in RH_\infty$, $Q_2(s) \in RH_\infty$, $Q_3(s) \in RH_\infty$ and $\alpha \in R$ is positive, (2.22), (2.23), (2.24) and (2.25) are all stable. Thus, the sufficiency has been shown.

We have thus proved Theorem 1. ∎

**Remark 2.3.1** *Semistrongly stabilizable plants in (2.3) have five parameters, and so appear complicated. However, the problem that $G(s)$ is semistrongly stabilizable is equivalent to the problem that $(s+\alpha)G(s)/s$ is strongly stabilizable. Therefore, while the equation (2.3) seems complicated, its meaning is simple.*

## 2.4 Numerical example

In this section, a numerical example is illustrated to show that the plant written by (2.3) is stabilizable by using semistrongly stabilizing controllers.

Consider the problem to design a semistrongly stabilizing controller $C(s)$ for the plant $G(s)$ written by:

$$G(s) = \frac{14s^2+90s+36}{s^3+9s^2+7s-1}. \tag{2.26}$$

Because $G(s)$ can be written by the form of (2.3), where $\alpha = 1$, $\beta = 6$,

$$Q_1(s) = \frac{1}{s+6}, \tag{2.27}$$

$$Q_2(s) = \frac{8}{s+1} \tag{2.28}$$

and

$$Q_3(s) = \frac{1}{s+6}, \tag{2.29}$$

$G(s)$ in (2.26) is semistrongly stabilizable. A semistrongly stabilizing controller $C(s)$ is given by:

$$C(s) = \frac{s+1}{s(s+6)}. \tag{2.30}$$

Using the semistrongly stabilizing controller $C(s)$ in (2.30), the response of the output $y(t)$ of the control system in (2.1) for the step reference input $r(t) = 1$ is shown in Fig. 2.1 . Figure 2.1 shows that the control system in (2.1) is stabilized by semistrongly stabilizing controller $C(s)$ in (2.30) and that the output $y(t)$ follows the step reference input $r(t)$ without steady state error.

In this way, we find that the plant written by the form of (2.3) is stabilizable by using semistrongly stabilizing controllers.

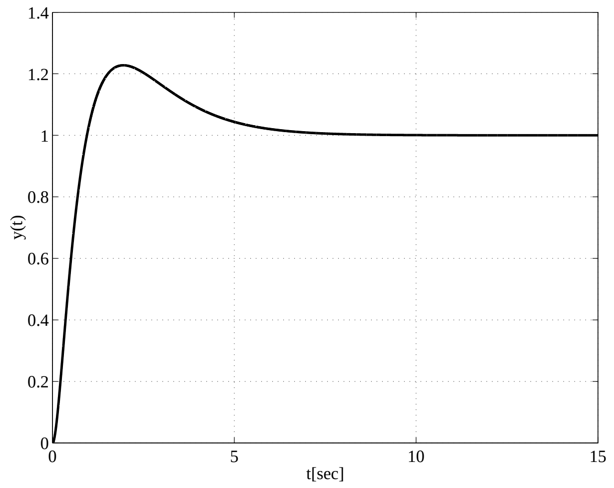


Fig. 2.1: Response of the output $y(t)$ of the control system in (2.1) for the step reference input $r(t) = 1$

## 2.5 Conclusion

In this chapter, the concept of semistrongly stabilizing controllers and semistrongly stabilizable plants was proposed. In addition, the parameterization of all semistrongly stabilizable plants was clarified. That is, it was shown that if the plant $G(s)$ is written by the form of (2.3), the plant is stabilizable by semistrongly stabilizing controllers. Finally, a numerical example was shown to illustrate that the plant written by (2.3) is stabilizable by using semistrongly stabilizable controllers.

# Chapter 3

# The parameterization of all semistrongly stabilizing controllers

## 3.1 Introduction

The semistrong stabilization is a control method to stabilize control system by a stabilizing controller that has one origin pole and other poles in the open left-half plane. The semistrong stabilization solves the problem in the strong stabilization that the output of the control system cannot follow the step reference input without steady state error in the presence of an uncertainty in the plant or a step disturbance.

In Chapter 2, semistrongly stabilizable plants are examined. Next, semistrongly stabilizing controllers are examined. From the definition and numerical example in Chapter 2, the semistrongly stabilizing controller in Chapter 2 is decided by plant parameters. From the viewpoint of control, this is undesirable because the controller design cannot be arbitrary. Therefore, the flexibility of semistrongly stabilizing controller design must be clarified.

One way to design the stabilizing controller with flexibility is to use the parameterization of all stabilizing controllers proposed by Youla et al. [1]. Because the semistrongly stabilizing controller is one of the stabilizing controllers, the semistrongly stabilizing controller is included in that parameterization. The controller design using that parameterization needs parameterized plant, and the parameterization of all semistrongly stabilizable plants is clarified in Chapter 2. Therefore, using these parameterizations, there is a possibility to be able to clarify the parameterization of all semistrongly stabilizing controllers. From this, it is expected that the design method for semistrongly stabilizing controllers with flexibility is also clarified.

In this chapter, we propose the parameterization of all semistrongly stabilizing controllers for the semistrongly stabilizable plants clarified in [28]. The control characteristics of the control system using the parameterization of all semistrongly stabilizing controllers are described. A design procedure for semistrongly stabilizing controllers is presented. A numerical example is presented to illustrate the effectiveness of the proposed method.

## 3.2 Problem formulation

Consider the control system:

$$\left\{ \begin{array}{rcl} y(s) & = & G(s)u(s) + d(s) \\ u(s) & = & C(s)\left(r(s) - y(s)\right) \end{array} \right. , \tag{3.1}$$

where $G(s) \in R(s)$ is the plant, $C(s) \in R(s)$ is the controller, $y(s)$ is the output, $u(s)$ is the control input, $d(s)$ is the disturbance, and $r(s)$ is the reference input.

In Chapter 2, the semistrongly stabilizing controller is composed by $\alpha$ and $Q_1(s)$, which are in semistrongly stabilizable plants in (2.3). Therefore, there is no freedom to design the semistrongly stabilizing controller in Chapter 2.

According to [1], the parameterization of all stabilizing controllers is written by:

$$C(s) = \frac{X(s) + D(s)P(s)}{Y(s) - N(s)P(s)}, \tag{3.2}$$

where $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ are coprime factors of $G(s)$ on $RH_\infty$ satisfying:

$$G(s) = \frac{N(s)}{D(s)}, \tag{3.3}$$

$X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are any functions satisfying:

$$N(s)X(s) + D(s)Y(s) = 1 \tag{3.4}$$

and $P(s) \in RH_\infty$ is any function. Because the semistrongly stabilizing controller is one of the stabilizing controllers, the semistrongly stabilizing controller is in the set of (3.9). Therefore, using this parameterization with the parameterization of all semistrongly stabilizable plants in Chapter 2, there is a possibility to be able to clarify the parameterization of all semistrongly stabilizing controllers. From this, it is expected that the design method for semistrongly stabilizing controllers with flexibility is also clarified.

In this chapter, we clarify the parameterization of all semistrongly stabilizing controllers for the strongly stabilizable plants $G(s)$ in (2.3).

## 3.3 The parameterization of all semistrongly stabilizing controllers for semistrongly stabilizable plants

In this section, the parameterization of all semistrongly stabilizing controllers $C(s)$ for the semistrongly stabilizable plant $G(s)$ in (2.3) is proposed.

This parameterization is summarized in the following theorem.

**Theorem 2** *The controller $C(s)$ is a semistrongly stabilizing controller for the semistrongly stabilizable plant $G(s)$ in (2.3) if and only if $C(s)$ is given by:*

$$C(s) = \frac{Q_1(s) + (1 + Q_3(s) - Q_1(s)Q_2(s))\, P(s)}{\dfrac{s}{s+\alpha} - \left(\dfrac{\beta}{s+\alpha} + \dfrac{sQ_2(s)}{s+\alpha}\right) P(s)}, \tag{3.5}$$

*where $P(s)$ is given by:*

$$P(s) = \frac{s}{s+\alpha} Q(s), \tag{3.6}$$

*$Q(s) \in RH_\infty$ is given by:*

$$Q(s) = \frac{1 - \hat{Q}(s)}{\dfrac{\beta}{s+\alpha} + \dfrac{s}{s+\alpha} Q_2(s)}, \tag{3.7}$$

*$\hat{Q}(s) \in \mathcal{U}$ is any function that makes $Q(s)$ in (3.7) proper and satisfies:*

$$\left.\frac{1}{(s - s_i)^{m_i - 1}} \left(1 - \hat{Q}(s)\right)\right|_{s=s_i} = 0 \quad (\forall i = 1, \ldots, n), \tag{3.8}$$

*$s_i (i = 1, \ldots, n)$ are unstable zeros of $\beta + sQ_2(s)$, and the multiplicities of $s_i (i = 1, \ldots, n)$ are denoted by $m_i (i = 1, \ldots, n)$.*

*(Proof)* From [12], the parameterization of all stabilizing controllers for $G(s)$, which are not necessarily semistrongly stabilizing controllers, is given by:

$$C(s) = \frac{X(s) + D(s)P(s)}{Y(s) - N(s)P(s)}, \tag{3.9}$$

where $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ are coprime factors of $G(s)$ on $RH_\infty$ satisfying:

$$G(s) = \frac{N(s)}{D(s)}, \tag{3.10}$$

$X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are any functions satisfying:

$$N(s)X(s) + D(s)Y(s) = 1 \tag{3.11}$$

and $P(s) \in RH_\infty$ is any function. Because the semistrongly stabilizable plant $G(s)$ takes the form of (2.3), $G(s)$ is factorized by (3.10), where:

$$N(s) = \frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s) \tag{3.12}$$

and

$$D(s) = 1 + Q_3(s) - Q_1(s)Q_2(s). \tag{3.13}$$

From (3.12) and (3.13), a pair of $X(s)$ and $Y(s)$ satisfying (3.11) is given by:

$$X(s) = Q_1(s) \tag{3.14}$$

and

$$Y(s) = \frac{s}{s+\alpha}. \tag{3.15}$$

Substituting (3.12), (3.13), (3.14), and (3.15) for (3.9), we have (3.5), where $P(s) \in RH_\infty$ is any function.

We now show that $C(s)$ in (3.5) is a semistrongly stabilizing controller if and only if $P(s)$ in (3.5) is given by (3.6), $Q(s)$ in (3.6) is given by (3.7), and $\hat{Q}(s)$ in (3.7) satisfies $\hat{Q}(s) \in \mathcal{U}$ and (3.8).

To prove necessity, we show that if $C(s)$ in (3.5) is a semistrongly stabilizing controller, then $P(s)$ in (3.5) is given by (3.6), $Q(s)$ in (3.5) is given by (3.7), and $\hat{Q}(s)$ in (3.7) satisfies $\hat{Q}(s) \in \mathcal{U}$ and (3.8). From the assumption that $C(s)$ in (3.5) is a semistrongly stabilizing controller and (3.5):

$$\left.\frac{s}{s+\alpha} - \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)P(s)\right|_{s=0} = 0 \tag{3.16}$$

is satisfied. This equation yields:

$$P(0) = 0. \tag{3.17}$$

This equation implies that $P(s)$ is given by (3.6), where $Q(s) \in RH_\infty$. Substituting (3.6) and (2.5) for (3.9), (3.9) is rewritten as:

$$C(s) = \frac{s+\alpha}{s}\left\{Q_1(s) + \frac{Q(s)}{1 - \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)Q(s)}\right\}. \tag{3.18}$$

From the assumption that $C(s)$ in (3.5) is a semistrongly stabilizing controller,

$$\begin{aligned}\bar{C}(s) &= \frac{s}{s+\alpha}C(s) \\ &= Q_1(s) + \frac{Q(s)}{1-\left(\frac{\beta}{s+\alpha}+\frac{s}{s+\alpha}Q_2(s)\right)Q(s)}\end{aligned} \tag{3.19}$$

must be included in $RH_\infty$. Because $Q_1(s) \in RH_\infty$ and $Q(s) \in RH_\infty$, the condition of $\bar{C}(s) \in RH_\infty$ in (3.19) is equivalent to:

$$1-\left(\frac{\beta}{s+\alpha}+\frac{s}{s+\alpha}Q_2(s)\right)Q(s) \in \mathcal{U}. \tag{3.20}$$

Using $\hat{Q}(s) \in \mathcal{U}$, let

$$1-\left(\frac{\beta}{s+\alpha}+\frac{s}{s+\alpha}Q_2(s)\right)Q(s) = \hat{Q}(s). \tag{3.21}$$

Equation (3.21) corresponds to (3.7). Because $s_i (i = 1, \ldots, n)$ are unstable zeros of $\beta + sQ_2(s)$ and the multiplicities of $s_i (i = 1, \ldots, n)$ are denoted by $m_i (i = 1, \ldots, n)$,

$$\left.\frac{1}{(s-s_i)^{m_i-1}}\left(\frac{\beta}{s+\alpha}+\frac{s}{s+\alpha}Q_2(s)\right)Q(s)\right|_{s=s_i} = 0 \quad (\forall i = 1, \ldots, n) \tag{3.22}$$

holds true. From (3.21) and (3.22), (3.8) is satisfied. Thus, the necessity has been shown.

Next, to prove sufficiency, we show that if $Q(s)$ in (3.5) is given by (3.7) and $\hat{Q}(s)$ in (3.7) satisfies $\hat{Q}(s) \in \mathcal{U}$ and (3.8), then $C(s)$ in (3.5) is a semistrongly stabilizing controller. Substituting (3.7) for (3.5), we have:

$$\begin{aligned}C(s) &= \frac{s+\alpha}{s}\left\{Q_1(s) + \frac{1-\hat{Q}(s)}{\left(\frac{\beta}{s+\alpha}+\frac{s}{s+\alpha}Q_2(s)\right)\hat{Q}(s)}\right\} \\ &= \frac{s+\alpha}{s}\left\{Q_1(s) + \frac{Q(s)}{\hat{Q}(s)}\right\}.\end{aligned} \tag{3.23}$$

Because $\hat{Q}(s) \in \mathcal{U}$ and $Q_1(s) \in RH_\infty$, if $Q(s) \in RH_\infty$, then $C(s)$ in (3.23) has a pole at the origin and other poles in the open left-half plane. Therefore, we show that $Q(s) \in RH_\infty$. From $\hat{Q}(s) \in \mathcal{U}$, if $Q(s)$ in (3.7) is unstable, unstable poles of $Q(s)$ are equal to unstable zeros $s_i (i = 1, \ldots, n)$ of $\beta + sQ_2(s)$. Because $\hat{Q}(s)$ satisfies (3.8), unstable zeros $s_i (i = 1, \ldots, n)$ of $\beta + sQ_2(s)$ are not equal to unstable poles of $Q(s)$. Therefore, $Q(s)$ is stable. In addition, $\hat{Q}(s)$ is selected to make $Q(s)$ in (3.7) proper, and $Q(s)$ in (3.7) satisfies $Q(s) \in RH_\infty$. Thus, $C(s)$ in (3.23) has a pole at the origin and other poles in the open left-half plane.

Next, we show that $C(s)$ in (3.23) makes the control system in (3.1) stable. By simple manipulation, we have:

$$\frac{G(s)C(s)}{1+G(s)C(s)} = 1 - \frac{s}{s+\alpha}\hat{Q}(s)\left(1+Q_3(s)-Q_1(s)Q_2(s)\right), \tag{3.24}$$

$$\frac{G(s)}{1+G(s)C(s)} = \frac{s\left(\beta+sQ_2(s)\right)}{(s+\alpha)^2}\hat{Q}(s), \tag{3.25}$$

$$\frac{C(s)}{1+G(s)C(s)} = (1+Q_3(s)-Q_1(s)Q_2(s))\left(Q_1(s)\hat{Q}(s)+Q(s)\right) \tag{3.26}$$

and

$$\frac{1}{1+G(s)C(s)} = \frac{s}{s+\alpha}\left(1+Q_3(s)-Q_1(s)Q_2(s)\right)\hat{Q}(s). \tag{3.27}$$

Because $\alpha > 0$, $Q_1(s) \in RH_\infty$, $Q_2(s) \in RH_\infty$, $Q_3(s) \in RH_\infty$, $\hat{Q}(s) \in \mathcal{U}$, and $Q(s) \in RH_\infty$, the transfer functions in (3.24), (3.25), (3.26), and (3.27) are stable. This implies that the control system in (3.1) is stable.

We have thus proved Theorem 2. ∎

Next, control characteristics of the control system in (3.1) using the parameterization of all semistrongly stabilizing controllers in (3.1) are explained.

The transfer functions from the reference input $r(s)$ to the output $y(s)$ and from the disturbance $d(s)$ to the output $y(s)$ of the control system in (3.1) are written as:

$$\frac{y(s)}{r(s)} = 1 - \frac{s}{s+\alpha}\hat{Q}(s)\left(1+Q_3(s)-Q_1(s)Q_2(s)\right) \tag{3.28}$$

and

$$\frac{y(s)}{d(s)} = \frac{s}{s+\alpha}\left(1+Q_3(s)-Q_1(s)Q_2(s)\right)\hat{Q}(s), \tag{3.29}$$

respectively. Therefore, using a semistrongly stabilizing controller $C(s)$ in (3.5), the output $y(s)$ follows the step reference input $r(s) = 1/s$ without steady state error and the step disturbance $d(s) = 1/s$ is attenuated effectively.

## 3.4 Design method for $\hat{Q}(s)$

From Theorem 2, to design a semistrongly stabilizing controller $C(s)$, $\hat{Q}(s)$ in (3.7) must be designed to be $\hat{Q}(s) \in \mathcal{U}$ to satisfy (3.8) and to make $Q(s)$ in (3.7) proper. In this section, a design method to ensure that $\hat{Q}(s) \in \mathcal{U}$ has these characteristics is presented.

The design method is summarized as follows.

1. We factorize:

   $$\tilde{Q}(s) = \frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)$$

   as

   $$\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s) = \tilde{Q}_i(s)\tilde{Q}_o(s), \tag{3.30}$$

   where $\tilde{Q}_i(s) \in RH_\infty$ is an inner function satisfying $\tilde{Q}_i(0) = 1$ and $\tilde{Q}_o(s) \in RH_\infty$ is an outer function.

2. Using $\tilde{Q}_o(s)$, we make $\bar{Q}(s) \in RH_\infty$:

   $$\bar{Q}(s) = \frac{q(s)}{\tilde{Q}_o(s)}, \tag{3.31}$$

   where:

   $$q(s) = \frac{k}{(\tau s+1)^m}, \tag{3.32}$$

   $\tau \in R$ is an arbitrary positive number, $m$ is an arbitrary positive integer to make $\bar{Q}(s)$ proper, and $k \in R$ is a real number satisfying $0 < k < 1$.

3. Using $\bar{Q}(s)$, $\hat{Q}(s) \in \mathcal{U}$ is designed as:

$$\hat{Q}(s) = 1 - \left( \frac{\beta}{s+\alpha} + \frac{s}{s+\alpha} Q_2(s) \right) \bar{Q}(s). \tag{3.33}$$

Next, we show that $\hat{Q}(s)$ in (3.33) satisfies $\hat{Q}(s) \in \mathcal{U}$ and (3.8), and makes $Q(s)$ in (3.7) proper. First, we show that $\hat{Q}(s)$ in (3.33) satisfies $\hat{Q}(s) \in \mathcal{U}$ and (3.8). Substituting (3.31) for (3.33), $\hat{Q}(s)$ in (3.33) is rewritten as:

$$\hat{Q}(s) = 1 - \tilde{Q}_i(s)q(s). \tag{3.34}$$

Because $\tilde{Q}_i(s)$ is an inner function, $\tilde{Q}_i(s)$ is biproper. That is, $\tilde{Q}_i(s)q(s)$ is strictly proper. In addition, from (3.32) and $0 < k < 1$,

$$\left\| \tilde{Q}_i(s)q(s) \right\|_\infty < 1. \tag{3.35}$$

This implies that $\hat{Q}(s) \in \mathcal{U}$.

Next, we show that (3.8) holds true. Because $s_i (i = 1, \ldots, n)$ are unstable zeros of $\beta + sQ_2(s)$, $m_i (i = 1, \ldots, n)$ denotes the multiplicities of $s_i (i = 1, \ldots, n)$, and $\tilde{Q}_i(s)$ is an inner function of $\beta/(s + \alpha) + sQ_2(s)/(s + \alpha)$,

$$\left. \frac{1}{(s - s_i)^{m_i - 1}} \tilde{Q}_i(s) \right|_{s = s_i} = 0 \quad (\forall i = 1, \ldots, n) \tag{3.36}$$

holds true. From this equation and (3.32),

$$\left. \frac{1}{(s - s_i)^{m_i - 1}} \tilde{Q}_i(s)q(s) \right|_{s = s_i} = 0 \quad (\forall i = 1, \ldots, n) \tag{3.37}$$

are also satisfied. From (3.34) and (3.37), $\hat{Q}(s)$ in (3.33) satisfies (3.8). Next, we show that $\hat{Q}(s)$ in (3.33) makes $Q(s)$ proper. Substituting (3.34) for (3.7), $Q(s)$ in (3.7) is rewritten as:

$$Q(s) = \bar{Q}(s). \tag{3.38}$$

Because $\bar{Q}(s) \in RH_\infty$, $Q(s)$ is proper. Therefore, $\hat{Q}(s)$ in (3.33) makes $Q(s)$ proper. Thus, we have shown that, using the method described above, we can design $\hat{Q}(s) \in \mathcal{U}$ to satisfy (3.8) and make $Q(s)$ in (3.7) proper.

## 3.5 Numerical example

In this section, a numerical example is presented to show the effectiveness of the proposed parameterization of all semistrongly stabilizing controllers for semistrongly stabilizable plants.

Consider the problem of designing a semistrongly stabilizing controller $C(s)$ for the angular velocity control of the two-inertia system in Fig. 3.1 . Here, $\tau_M$ is the torque of the motor, $J_M$ is the moment of inertia of the motor, $D_M$ is the coefficient of friction of the motor, $J_L$ is the moment of inertia of the load, $D_L$ is the coefficient of friction of the load, $K$ is the torsional spring constant, and $\omega_L$ is the angular velocity of the load. For our example, we use the values $J_M = 2.0 \cdot 10^{-4}$, $D_M = 0.8 \cdot 10^{-3}$, $J_L = 2.2 \cdot 10^{-2}$, $D_L = 1.8 \cdot 10^{-3}$, and $K = 0.4$. This plant is then given by:

$$G(s) = \frac{90.9 \cdot 10^3}{(s + 0.117)(s^2 + 3.97s + 2.02 \cdot 10^3)}. \tag{3.39}$$

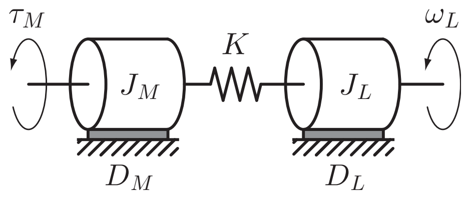


Fig. 3.1: Two-inertia system

First, we show that the plant $G(s)$ in (3.39) can be rewritten in the form of (2.3). $\alpha \in R$ is set to:

$$\alpha = 1 \tag{3.40}$$

and $Q_1(s) \in RH_\infty$ is set to:

$$Q_1(s) = 0.26 \cdot 10^{-2}. \tag{3.41}$$

Substituting (3.40) and (3.41) for (2.4) and (2.5), $\beta \in R$ is given by:

$$\beta = 3.85 \cdot 10^2 \tag{3.42}$$

and $Q_3(s) \in RH_\infty$ is given by:

$$Q_3(s) = 0. \tag{3.43}$$

Therefore, $Q_2(s) \in RH_\infty$ is given by:

$$Q_2(s) = -\frac{3.85 \cdot 10^2(s^2 + 4.08s + 1.78 \cdot 10^3)}{(s^2 + 0.234s + 0.117)(s^2 + 3.85s + 2.02 \cdot 10^3)}. \tag{3.44}$$

Using (3.40), (3.41), (3.42), (3.44), and (3.43), the plant $G(s)$ in (3.39) is rewritten in the form of (2.3). That is, $G(s)$ in (3.39) is semistrongly stabilizable.

For the plant $G(s)$ in (3.39), we design a semistrongly stabilizing controller. $\hat{Q}(s) \in \mathcal{U}$ in (3.7) must satisfy (3.8) and make $Q(s)$ in (3.7) proper. Using the method in Section 4.4, we design $\hat{Q}(s)$. $\tilde{Q}(s)$ in (3.30) is factorized by (3.30), where:

$$\tilde{Q}_i(s) = 1 \tag{3.45}$$

and

$$\tilde{Q}_o(s) = \frac{90.9 \cdot 10^3(s+1)}{(s^2 + 0.234s + 0.117)(s^2 + 3.85s + 2.02 \cdot 10^3)}, \tag{3.46}$$

respectively. $\bar{Q}(s)$ is made (3.31), where $q(s)$ is given by (3.32),

$$\tau = 0.02, \tag{3.47}$$

$$m = 3, \tag{3.48}$$

and

$$k = 0.99, \tag{3.49}$$

respectively. Using this $\bar{Q}(s)$, $\hat{Q}(s)$ is set to (3.33). In summary, $\hat{Q}(s) \in \mathcal{U}$ becomes:

$$\hat{Q}(s) = \frac{(s+0.167)(s^2+1.50\cdot 10^2 s+7.48\cdot 10^3)}{(s+50)^3}. \tag{3.50}$$

We find that the designed $\hat{Q}(s)$ is a unimodular function. Substituting (3.50) and (3.7) for (3.5), we have a semistrongly stabilizing controller for the semistrongly stabilizable plant $G(s)$ in (3.39):

$$C(s) = \frac{1.36(s^2+0.241s+0.118)(s^2+4.12s+2.03\cdot 10^3)}{s(s+0.167)(s^2+1.50\cdot 10^2 s+7.48\cdot 10^3)}. \tag{3.51}$$

It is obvious that $C(s)$ in (3.51) has a pole at the origin and other poles in the open left-half plane, that is, $C(s)$ in (3.51) is a semistrongly stabilizing controller for $G(s)$ if $C(s)$ in (3.51) stabilizes $G(s)$ in (3.39).

Using this semistrongly stabilizing controller $C(s)$ in (3.51), the response of the output $y(t)$ of the control system in (3.1) for the step reference input $r(t) = 1$ is shown in Fig. 3.2 . Figure

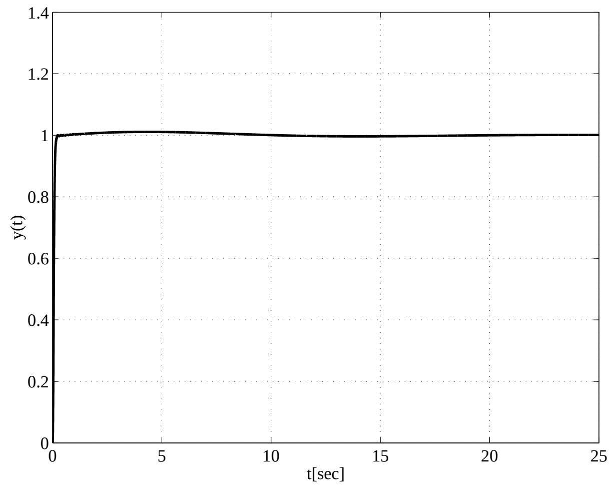


Fig. 3.2: Response of the output $y(t)$ of the control system in (3.1) for the step reference input $r(t) = 1$

3.2 shows that the control system in (3.1) is stable and the output $y(t)$ follows the step reference input $r(t) = 1$ without steady state error.

We have thus confirmed that the controller designed using the method in Section 4.4 is a semistrongly stabilizing controller. In addition, we have also confirmed that we can design semistrongly stabilizing controllers systematically by considering the angular velocity control of our two-inertia system, which is a real application.

## 3.6 Conclusions

In this chapter, the parameterization of all semistrongly stabilizing controllers for semistrongly stabilizable plants was clarified. The control characteristic using semistrongly stabilizable plants was presented. A design method for $\hat{Q}(s) \in \mathcal{U}$ that satisfies (3.8) and makes $Q(s)$ proper was also presented. In addition, a numerical example was presented and the effectiveness of the proposed method was illustrated.

# Chapter 4

# The parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers

## 4.1 Introduction

The semistrong stabilization is a control method to stabilize control system by a stabilizing controller that has one origin pole and other poles in the open left-half plane. The semistrong stabilization solves the problem in the strong stabilization that the output of the control system cannot follow the step reference input without steady state error in the presence of an uncertainty in the plant or a step disturbance.

In Chapter 3, the parameterization of all semistrongly stabilizing controllers [29] is clarified. This parameterization solves the problem that the semistrongly stabilizing controller in Chapter 2 has no flexibility to design. This result enables us to obtain the semistrongly stabilizing controller that guarantees the stability of control system and can design for the required control performance.

However, with their parameterization [29], we cannot specify the input–output characteristic and the feedback characteristic, that is, a disturbance attenuation characteristic and robust stability, separately. When we specify one characteristic, other characteristics are also decided. From the practical viewpoint, it is desirable to specify the input–output characteristic and the feedback characteristic separately. One way to achieve this is to use a two-degrees-of-freedom control system. In addition, because a two-degrees-of-freedom control system can have no overshoot for the reference input, more accurate control can be expected.

In this chapter, we propose the parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers for semistrongly stabilizable plants.

## 4.2 Two-degrees-of-freedom semistrongly stabilizing controller and problem formulation

Consider the two-degrees-of-freedom control system shown in Fig. 4.1 , which can specify the input–output characteristic and the feedback characteristic separately. Here, $G(s) \in R(s)$ is the plant, $C(s)$ is the two-degrees-of-freedom controller:

$$C(s) = \begin{bmatrix} C_1(s) & -C_2(s) \end{bmatrix}, \tag{4.1}$$

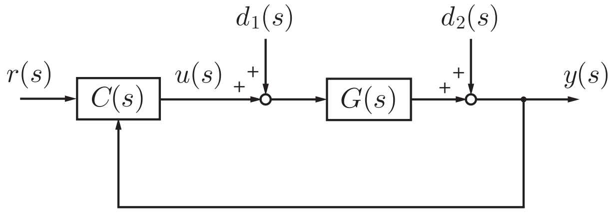


Fig. 4.1: Two-degrees-of-freedom control system

$u(s)$ is the control input:

$$u(s) = C(s) \begin{bmatrix} r(s) \\ y(s) \end{bmatrix} = \begin{bmatrix} C_1(s) & -C_2(s) \end{bmatrix} \begin{bmatrix} r(s) \\ y(s) \end{bmatrix}, \tag{4.2}$$

$r(s)$ is the reference input, $d_1(s)$ and $d_2(s)$ are disturbances, and $y(s)$ is the output. In the following, we call $C_1(s) \in R(s)$ the feed-forward controller and $C_2(s) \in R(s)$ the feedback controller. From the definition of internal stability [12], when all transfer functions $V_i(s)(i = 1, \ldots, 6)$:

$$\begin{bmatrix} u(s) \\ y(s) \end{bmatrix} = \begin{bmatrix} V_1(s) & V_2(s) & V_3(s) \\ V_4(s) & V_5(s) & V_6(s) \end{bmatrix} \begin{bmatrix} r(s) \\ d_1(s) \\ d_2(s) \end{bmatrix} \tag{4.3}$$

are stable, the two-degrees-of-freedom control system in Fig. 4.1 is stable.

Semistrongly stabilizing controllers have been defined in Chapter 2 as follows.

**Definition 3** *(semistrongly stabilizing controllers)[28]*
*We call the controller $C(s)$ a "semistrongly stabilizing controller" if the stabilizing controller has only one pole at the origin and other poles in the open left-half plane. That is, if $C(s)$ is:*

$$C(s) = \frac{s+\alpha}{s} Q(s), \tag{4.4}$$

*then we call $C(s)$ a semistrongly stabilizing controller, where $\alpha \in R$ is any positive real number and $Q(s) \in RH_\infty$ is any function satisfying $Q(0) \neq 0$.*

According to Definition 3, the difference between strongly stabilizing controllers and semistrongly stabilizing controllers is whether or not the controllers have only one pole at the origin. That is, the characteristic of semistrongly stabilizing controllers is to make the output of the control system follow the step reference input without steady state error in the presence of an uncertainty in the plant or a step disturbance. In addition, a plant stabilizable by a semistrongly stabilizing controller, so-called semistrongly stabilizable plants, have been also defined in Chapter 2 as follows.

**Definition 4** *(semistrongly stabilizable plant)[28]*
*We call the plant $G(s)$ a "semistrongly stabilizable plant" if $G(s)$ is stabilizable by a semistrongly stabilizing controller $C(s)$ in (4.4).*

According to Chapter 2, the parameterization of all semistrongly stabilizable plants [28] is defined by:

$$G(s) = \frac{\beta + sQ_2(s)}{(s+\alpha)\left(1 + Q_3(s) - Q_1(s)Q_2(s)\right)}, \tag{4.5}$$

where $\beta \in R$ is given by:

$$\beta = \frac{\alpha}{Q_1(0)}, \tag{4.6}$$

$Q_3(s) \in RH_\infty$ is given by:

$$Q_3(s) = \frac{\alpha - \beta Q_1(s)}{s}, \tag{4.7}$$

and $Q_1(s) \in RH_\infty$ and $Q_2(s) \in RH_\infty$ are any functions satisfying $Q_1(0) \neq 0$. That is, the plant $G(s)$ in Fig. 4.1 is described by the form of (4.5). In addition, Hoshikawa et al. have given the parameterization of all semistrongly stabilizing controllers for semistrongly stabilizable plants in (4.5) [29].

However, the parameterization of all semistrongly stabilizing controllers [29] was only considered for one-degree-of-freedom control systems. This means that all characteristics are specified for a one-degree-of-freedom semistrongly stabilizing controller. From the practical point of view, it is desirable to specify the input–output characteristic and the feedback characteristic separately. One way to do this is to use a two-degrees-of-freedom control system in Fig. 4.1 .

From this viewpoint, we consider a two-degrees-of-freedom semistrongly stabilizing controller that makes the output of the control system follow a step reference input without steady state error, under the existence of an uncertainty in the plant or a step disturbance. The concept of a two-degrees-of-freedom semistrongly stabilizing controller is proposed as follows.

**Definition 5** *(two-degrees-of-freedom semistrongly stabilizing controller)*
*We call the controller $C(s)$ in (4.1) a "two-degrees-of-freedom semistrongly stabilizing controller" if the following expressions hold true.*

1. *The feed-forward controller $C_1(s)$ in (4.1) has only one pole at the origin. That is, the feed-forward controller $C_1(s)$ is defined by:*

$$C_1(s) = \frac{s+\gamma}{s} Q_f(s), \tag{4.8}$$

   *where $\gamma \in R$ is any positive real number and $Q_f(s) \in RH_\infty$ is any function satisfying $Q_f(0) \neq 0$.*

2. *The feedback controller $C_2(s)$ in (4.1) works as a semistrongly stabilizing controller. That is, the feedback controller $C_2(s)$ is defined in the form of (4.4).*

3. *The two-degrees-of-freedom control system in Fig. 4.1 is stable. That is, all transfer functions $V_i(s)(i = 1, \ldots, 6)$ in (4.3) are stable.*

From Definition 5, the feed-forward controller $C_1(s)$ also has a pole at the origin. This means the transfer function $V_4(s)$ in (4.3) from the reference input $r(s)$ to the output $y(s)$ in Fig. 4.1 cannot have a zero at the origin with the origin pole of the feedback controller $C_2(s)$, which is to ensure that the output cannot have a steady state error for the step reference input.

The problem considered in this chapter is to obtain the parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers $C(s)$ defined in Definition 5.

## 4.3 The parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers

In this section, we propose the parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers $C(s)$ for semistrongly stabilizable plants $G(s)$ in the form of (4.5).

This parameterization is summarized in the following theorem.

**Theorem 3** *The parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers $C(s)$ for semistrongly stabilizable plants $G(s)$ in the form of (4.5) is:*

$$C_1(s) = \frac{s+\gamma}{s}\frac{Q_{c1}(s)}{Q_u(s)} \tag{4.9}$$

*and*

$$C_2(s) = \frac{s+\alpha}{s}\left\{Q_1(s) + \frac{Q_{c2}(s)}{1-\left(\frac{\beta}{s+\alpha}+\frac{s}{s+\alpha}Q_2(s)\right)Q_{c2}(s)}\right\}. \tag{4.10}$$

*Here, $\alpha \in R$ and $\gamma \in R$ are any positive real numbers, $Q_{c1}(s) \in RH_\infty$ is any function, $Q_{c2}(s) \in RH_\infty$ is given by:*

$$Q_{c2}(s) = \frac{1-Q_u(s)}{\dfrac{\beta}{s+\alpha}+\dfrac{s}{s+\alpha}Q_2(s)}, \tag{4.11}$$

*$Q_u(s) \in \mathcal{U}$ is any function that makes $Q_{c2}(s)$ in (4.11) proper and satisfies:*

$$\left.\frac{1}{(s-s_i)^{m_i-1}}\left(1-Q_u(s)\right)\right|_{s=s_i} = 0 \quad (\forall i = 1,\dots,n), \tag{4.12}$$

*$s_i(i=1,...,n)$ are unstable zeros of $\beta + sQ_2(s)$, and the multiplicities of $s_i(i=1,...,n)$ are denoted by $m_i(i=1,...,n)$.*

*(Proof)* First, the necessity is shown. That is, we show that if the controllers $C_1(s)$ and $C_2(s)$ make the control system in Fig. 4.1 stable, that is all transfer functions $V_i(s)(i=1,\dots,6)$ in (4.3) are stable, then $C_1(s)$ and $C_2(s)$ are defined by (4.9) and (4.10), respectively. The transfer functions $V_i(s)(i=1,\dots,6)$ in (4.3) are:

$$V_1(s) = \frac{C_1(s)}{1+C_2(s)G(s)}, \tag{4.13}$$

$$V_2(s) = -\frac{C_2(s)G(s)}{1+C_2(s)G(s)}, \tag{4.14}$$

$$V_3(s) = -\frac{C_2(s)}{1+C_2(s)G(s)}, \tag{4.15}$$

$$V_4(s) = \frac{C_1(s)G(s)}{1+C_2(s)G(s)}, \tag{4.16}$$

$$V_5(s) = \frac{G(s)}{1+C_2(s)G(s)}, \tag{4.17}$$

and

$$V_6(s) = \frac{1}{1+C_2(s)G(s)}. \tag{4.18}$$

From the assumption that all transfer functions in (4.13) to (4.18) are stable, $C_2(s)$ makes $G(s)$ stable. From [12], the parameterization of all stabilizing feedback controllers is:

$$C_2(s) = \frac{X(s) + D(s)\tilde{Q}(s)}{Y(s) - N(s)\tilde{Q}(s)}, \tag{4.19}$$

where $N(s)$ and $D(s)$ are coprime factors of $G(s)$ on $RH_\infty$ satisfying:

$$G(s) = \frac{N(s)}{D(s)}, \tag{4.20}$$

$X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are any functions satisfying:

$$N(s)X(s) + D(s)Y(s) = 1, \tag{4.21}$$

and $\tilde{Q}(s) \in RH_\infty$ is any function. Therefore, we must consider the condition to make $C_2(s)$ in (4.19) work as a semistrongly stabilizing controllers. Since the semistrongly stabilizable plant $G(s)$ is defined by the form of (4.5), when $G(s)$ in (4.5) is factorized by (4.20):

$$N(s) = \frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s) \tag{4.22}$$

and

$$D(s) = 1 + Q_3(s) - Q_1(s)Q_2(s). \tag{4.23}$$

From (4.22) and (4.23), a pair of $X(s)$ and $Y(s)$ satisfying (4.21) are defined by:

$$X(s) = Q_1(s) \tag{4.24}$$

and

$$Y(s) = \frac{s}{s+\alpha}. \tag{4.25}$$

Substituting (4.22), (4.23), (4.24) and (4.25) for (4.19), we have:

$$\begin{aligned} C_2(s) &= \frac{Q_1(s) + (1 + Q_3(s) - Q_1(s)Q_2(s))\,\tilde{Q}(s)}{\frac{s}{s+\alpha} - \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\tilde{Q}(s)} \\ &= \frac{s+\alpha}{s}\left\{\frac{Q_1(s) + (1 + Q_3(s) - Q_1(s)Q_2(s))\,\tilde{Q}(s)}{1 - \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\frac{s+\alpha}{s}\tilde{Q}(s)}\right\}. \end{aligned} \tag{4.26}$$

From the assumption that $C_2(s)$ has one pole at the origin, $\tilde{Q}(s)$ becomes:

$$\tilde{Q}(s) = \frac{s}{s+\alpha}Q_{c2}(s), \tag{4.27}$$

where $Q_{c2}(s) \in RH_\infty$ is any function. Substituting (4.7) and (4.27) for (4.26), we have:

$$C_2(s) = \frac{Q_1(s) + \left(1 + \frac{\alpha - \beta Q_1(s)}{s} - Q_1(s)Q_2(s)\right)\frac{s}{s+\alpha}Q_{c2}(s)}{\frac{s}{s+\alpha} - \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\frac{s}{s+\alpha}Q_{c2}(s)}. \tag{4.28}$$

By simple manipulation, we have:

$$C_2(s) = \frac{s+\alpha}{s}\left\{Q_1(s) + \frac{Q_{c2}(s)}{1-\left(\frac{\beta}{s+\alpha}+\frac{s}{s+\alpha}Q_2(s)\right)Q_{c2}(s)}\right\}. \tag{4.29}$$

We have therefore shown that $C_2(s)$ is defined by (4.10). The remaining problem is to confirm that:

$$\bar{C}_2(s) = Q_1(s) + \frac{Q_{c2}(s)}{1-\left(\frac{\beta}{s+\alpha}+\frac{s}{s+\alpha}Q_2(s)\right)Q_{c2}(s)} \tag{4.30}$$

is stable. Since $Q_1(s) \in RH_\infty$ and $Q_{c2}(s) \in RH_\infty$, the condition that (4.30) is stable if and only if $Q_{c2}(s)$ in (4.30) results in:

$$1-\left(\frac{\beta}{s+\alpha}+\frac{s}{s+\alpha}Q_2(s)\right)Q_{c2}(s) \in \mathcal{U}. \tag{4.31}$$

That is, using $Q_u(s) \in \mathcal{U}$,

$$1-\left(\frac{\beta}{s+\alpha}+\frac{s}{s+\alpha}Q_2(s)\right)Q_{c2}(s) = Q_u(s). \tag{4.32}$$

This equation corresponds to (4.11). Since $s_i(i=1,\ldots,n)$ denote unstable zeros of $\beta + sQ_2(s)$ and the multiplicities of $s_i(i=1,\ldots,n)$ are denoted by $m_i(i=1,\ldots,n)$,

$$\frac{1}{(s-s_i)^{m_i-1}}\left(\frac{\beta}{s+\alpha}+\frac{s}{s+\alpha}Q_2(s)\right)Q_{c2}(s)\bigg|_{s=s_i} = 0 \ \ (\forall i=1,\ldots,n) \tag{4.33}$$

hold true. From (4.32) and (4.33), (4.12) is satisfied. The fact that $Q_{c2}(s)$ in (4.11) is included in $RH_\infty$ is confirmed as follows: From $Q_u(s) \in \mathcal{U}$, if $Q_{c2}(s)$ in (4.11) is unstable, then unstable poles of $Q_{c2}(s)$ are equal to unstable zeros $s_i(i=1,\ldots,n)$ of $\beta + sQ_2(s)$. Since $Q_u(s)$ satisfies (4.12), unstable zeros $s_i(i=1,\ldots,n)$ of $\beta + sQ_2(s)$ are not equal to unstable poles of $Q_{c2}(s)$. Therefore, $(\beta/(s+\alpha)+sQ_2(s)/(s+\alpha))Q_{c2}(s)$ is stable. That is, when we select $Q_u(s)$ to make $Q_{c2}(s)$ proper, $Q_{c2}(s)$ in (4.11) is included in $RH_\infty$. In addition, using $Q_u(s)$, $\bar{C}_2(s)$ in (4.30) is rewritten:

$$\bar{C}_2(s) = Q_1(s) + \frac{Q_{c2}(s)}{Q_u(s)}. \tag{4.34}$$

Since $Q_1(s) \in RH_\infty$, $Q_{c2}(s) \in RH_\infty$ and $Q_u(s) \in \mathcal{U}$, $\bar{C}_2(s) \in RH_\infty$. In this way, the fact that $C_2(s)$ works as a semistrongly stabilizing controller in (4.4) is shown.

Next, we show that the feed-forward controller $C_1(s)$ is described by (4.9). Using $C_2(s)$ in (4.10), the transfer functions in (4.13) and (4.16) are:

$$V_1(s) = C_1(s)Q_u(s)\left\{1 - Q_1(s)\left(\frac{\beta}{s+\alpha}+\frac{s}{s+\alpha}Q_2(s)\right)\right\} \tag{4.35}$$

and

$$V_4(s) = \frac{s}{s+\alpha}C_1(s)Q_u(s)\left(\frac{\beta}{s+\alpha}+\frac{s}{s+\alpha}Q_2(s)\right), \tag{4.36}$$

respectively. From (4.6),

$$1 - Q_1(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\Big|_{s=0} = 0 \tag{4.37}$$

in (4.35) holds true. For $V_1(s)$ in (4.35) and $V_4(s)$ in (4.36) to be stable, $C_1(s) \in RH_\infty$ or $C_1(s)$ can have only one pole at the origin and has other poles in the open left-half plane. Therefore, $C_1(s)$ in (4.8) works as a stabilizing controller. To specify the input–output characteristic and the feedback characteristic separately, $C_1(s)$ in (4.8) becomes:

$$C_1(s) = \frac{s+\gamma}{s}\frac{Q_{c1}(s)}{Q_u(s)}, \tag{4.38}$$

where, $Q_{c1}(s) \in RH_\infty$ is any function. In this way, when the plant $G(s)$ takes the form of (4.5), then $C_1(s)$ and $C_2(s)$ take the form of (4.9) and (4.10). Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, we show that if $C_1(s)$ and $C_2(s)$ are described by (4.9) and (4.10), $C_1(s)$ and $C_2(s)$ make the control system in Fig. 4.1 stable. Using $C_1(s)$ in (4.9) and $C_2(s)$ in (4.10), transfer functions $V_i(s)(i = 1, \ldots, 6)$ are written:

$$V_1(s) = \frac{s+\gamma}{s}Q_{c1}(s)\left\{1 - Q_1(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\right\}, \tag{4.39}$$

$$V_2(s) = Q_u(s)\left\{1 - Q_1(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\right\} - 1, \tag{4.40}$$

$$V_3(s) = -\frac{s+\alpha}{s}\left(Q_1(s)Q_u(s) + Q_{c2}(s)\right)\left\{1 - Q_1(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\right\}, \tag{4.41}$$

$$V_4(s) = \frac{s+\gamma}{s+\alpha}Q_{c1}(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right), \tag{4.42}$$

$$V_5(s) = \frac{s}{s+\alpha}Q_u(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right), \tag{4.43}$$

and

$$V_6(s) = Q_u(s)\left\{1 - Q_1(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\right\}. \tag{4.44}$$

Since $Q_1(s) \in RH_\infty$, $Q_2(s) \in RH_\infty$, $Q_{c1}(s) \in RH_\infty$, $Q_u(s) \in \mathcal{U}$, and $\alpha$ is a positive real number, (4.40), (4.42), (4.43) and (4.44) are all stable. In addition, since (4.37) holds true, (4.39) and (4.42) have no pole at the origin. From this and because $Q_{c2}(s) \in RH_\infty$, (4.39) and (4.42) are also stable. Thus, the sufficiency has been shown.

We have thus proved Theorem 3.

∎

Next, we explain the control characteristics of the control system in Fig. 4.1 using the parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers in (4.9) and (4.10). First, the input–output characteristic is shown. The transfer function from the reference input $r(s)$ to the output $y(s)$ is:

$$\frac{y(s)}{r(s)} = \frac{s+\gamma}{s+\alpha}Q_{c1}(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right). \tag{4.45}$$

For the output $y(s)$ to follow the step reference input $r(s)$ without steady state error:

$$\frac{\gamma}{\alpha}\frac{\beta}{\alpha}Q_{c1}(0) = 1 \tag{4.46}$$

must be satisfied. From (4.6), (4.46) is rewritten:

$$\frac{\gamma}{\alpha}\frac{Q_{c1}(0)}{Q_1(0)} = 1. \tag{4.47}$$

Therefore, we select $Q_{c1}(s)$ satisfying:

$$Q_{c1}(0) = \frac{\alpha}{\gamma}Q_1(0). \tag{4.48}$$

Next, the disturbance attenuation characteristic, which is one of the feedback characteristics, is shown. The transfer functions from the disturbance $d_1(s)$ to the output $y(s)$ and from the disturbance $d_2(s)$ to the output $y(s)$ of the control system in Fig. 4.1 are:

$$\frac{y(s)}{d_1(s)} = \frac{s}{s+\alpha}Q_u(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right) \tag{4.49}$$

and

$$\frac{y(s)}{d_2(s)} = Q_u(s)\left\{1 - Q_1(s)\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\right\}, \tag{4.50}$$

respectively. Equation 4.49 shows that the step disturbance $d_1(s) = 1/s$ is attenuated effectively. In addition, since (4.37) holds true, the step disturbance $d_2(s) = 1/s$ is also attenuated effectively.

Furthermore, we find that the input–output characteristic is specified by $Q_{c1}(s)$ in (4.9), and the disturbance attenuation characteristic is specified by $Q_u(s)$ in (4.12). That is, the proposed two-degrees-of-freedom semistrongly stabilizing controller can specify the input–output characteristic and the disturbance attenuation characteristic separately.

## 4.4 Design method for $Q_u(s)$

In this section, we present a design method for $Q_u(s) \in \mathcal{U}$ that satisfies (4.12) and makes $Q_{c2}(s)$ proper.

1. $\beta/(s+\alpha) + sQ_2(s)/(s+\alpha)$ is factorized:

$$\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s) = Q_i(s)Q_o(s), \tag{4.51}$$

   where $Q_i(s) \in RH_\infty$ is the inner function satisfying $Q_i(0) = 1$ and $Q_o(s) \in RH_\infty$ is the outer function.

2. Using $Q_o(s)$, $\bar{Q}(s) \in RH_\infty$ is designed:

$$\bar{Q}(s) = \frac{q(s)}{Q_o(s)}, \tag{4.52}$$

   where

$$q(s) = \frac{k}{(\tau s + 1)^\epsilon}, \tag{4.53}$$

   $\tau \in R$ is an arbitrary positive number, $\epsilon$ is an arbitrary positive integer to make $\bar{Q}(s)$ proper, and $k \in R$ is a real number satisfying $0 < k < 1$.

3. Using $\bar{Q}(s)$, $Q_u(s) \in \mathcal{U}$ is designed:

$$Q_u(s) = 1 - \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\bar{Q}(s). \tag{4.54}$$

Next, we show that $Q_u(s)$ in (4.54) satisfies (4.12) and makes $Q_{c2}(s)$ proper. First, we show that $Q_u(s)$ in (4.54) satisfies (4.12). Substituting (4.52) for (4.54), $Q_u(s)$ in (4.54) is rewritten:

$$Q_u(s) = 1 - Q_i(s)q(s). \tag{4.55}$$

Since $s_i(i = 1, \ldots, n)$ are unstable zeros of $\beta + sQ_2(s)$, $m_i(i = 1, \ldots, n)$ denote multiplicities of $s_i(i = 1, \ldots, n)$, and $Q_i(s)$ is an inner function of $\beta/(s+\alpha) + sQ_2(s)/(s+\alpha)$:

$$\left.\frac{1}{(s-s_i)^{m_i-1}}Q_i(s)\right|_{s=s_i} = 0 \quad (\forall i = 1, \ldots, n) \tag{4.56}$$

holds true. From this equation and (4.53):

$$\left.\frac{1}{(s-s_i)^{m_i-1}}Q_i(s)q(s)\right|_{s=s_i} = 0 \quad (\forall i = 1, \ldots, n) \tag{4.57}$$

are also satisfied. From (4.55) and (4.57), $Q_u(s)$ in (4.54) satisfies (4.12). Next, we show that $Q_u(s)$ in (4.54) makes $Q_{c2}(s)$ proper. Substituting (4.54) for (4.11), $Q_{c2}(s)$ is rewritten:

$$Q_{c2}(s) = \bar{Q}(s). \tag{4.58}$$

Since $\bar{Q}(s) \in RH_\infty$, $Q_{c2}(s)$ is proper. Therefore, $Q_u(s)$ in (4.54) makes $Q_{c2}(s)$ proper.

## 4.5 Numerical example

We provide a numerical example to compare responses of a one-degree-of-freedom control system [29] and a two-degrees-of-freedom control system to show the effectiveness of the proposed method.

The plant considered in [29] is the angular velocity control of the two-inertia system in Fig. 4.2 . Here, $\tau_M$ is the torque of the motor, $J_M$ is the moment of inertia of the motor, $D_M$ is the

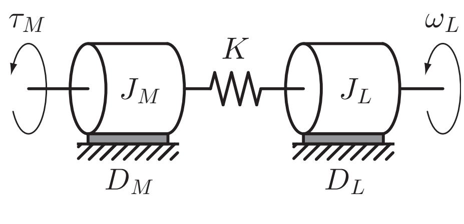


Fig. 4.2: Two-inertia system

coefficient of friction of the motor, $J_L$ is the moment of inertia of the load, $D_L$ is the coefficient of friction of the load, $K$ is the torsional spring constant, and $\omega_L$ is the angular velocity of the load. In [29], $J_M = 2.0 \cdot 10^{-4}$, $D_M = 0.8 \cdot 10^{-3}$, $J_L = 2.2 \cdot 10^{-2}$, $D_L = 1.8 \cdot 10^{-3}$, and $K = 0.4$. For this plant, we design a two-degrees-of-freedom semistrongly stabilizing controller, and contrast the responses of the one-degree-of-freedom control system in [29] and our two-degrees-of-freedom control system.

The plant in Fig. 4.2 is described by:

$$G(s) = \frac{90.9 \cdot 10^3}{(s+0.117)(s^2+3.97s+2.02 \cdot 10^3)}. \tag{4.59}$$

In [29], the plant $G(s)$ in (4.59) was rewritten in the form of (4.5). Here, $\alpha = 1$, $\beta = 3.85 \cdot 10^2$:

$$Q_1(s) = 0.26 \cdot 10^{-2}, \tag{4.60}$$

$$Q_2(s) = -\frac{3.85 \cdot 10^2(s^2+4.08s+1.78 \cdot 10^3)}{(s^2+0.234s+0.117)(s^2+3.85s+2.02 \cdot 10^3)}, \tag{4.61}$$

and

$$Q_3(s) = 0. \tag{4.62}$$

First, we design the feedback controller $C_2(s)$ in (4.10). To show that the feedback characteristics of the two-degrees-of-freedom control system can be equal to that of the one-degree-of-freedom control system, we set $C_2(s)$ equal to $C(s)$ in [29]. That is, $Q_i(s)$ and $Q_o(s)$ in (4.51) are:

$$\tilde{Q}_i(s) = 1 \tag{4.63}$$

and

$$\tilde{Q}_o(s) = \frac{90.9 \cdot 10^3(s+1)}{(s^2+0.234s+0.117)(s^2+3.85s+2.02 \cdot 10^3)}, \tag{4.64}$$

respectively. In addition, $\tau$, $\epsilon$, and $k \in R$ in (4.53) are set to $\tau = 0.02$, $\epsilon = 3$, and $k = 0.99$, respectively. Using these parameters, $Q_u(s)$ in (4.54) and $C_2(s)$ in (4.10) are given by:

$$Q_u(s) = \frac{(s+0.167)(s^2+1.50 \cdot 10^2 s+7.48 \cdot 10^3)}{(s+50)^3} \tag{4.65}$$

and

$$C_2(s) = \frac{1.36(s^2+0.241s+0.118)(s^2+4.12s+2.03 \cdot 10^3)}{s(s+0.167)(s^2+1.50 \cdot 10^2 s+7.48 \cdot 10^3)}, \tag{4.66}$$

respectively.

Next, we design the feed-forward controller $C_1(s)$ in (4.9). Since the transfer function from the reference input $r(s)$ to the output $y(s)$ is described by $V_4(s)$ in (4.42), $Q_{c1}(s)$ in (4.9) is designed as:

$$Q_{c1}(s) = \frac{s+\alpha}{s+\gamma}\frac{1}{(\tau_{c1}s+1)^{\epsilon_{c1}}}\frac{1}{\dfrac{\beta}{s+\alpha}+\dfrac{s}{s+\alpha}Q_2(s)}, \tag{4.67}$$

where $\tau_{c1} \in R$ is an arbitrary positive number and $\epsilon_{c1}$ is an arbitrary positive integer to make $Q_{c1}(s)$ proper. When $\gamma$, $\tau_{c1}$, and $\epsilon_{c1}$ are set to $\gamma = 1$, $\tau_{c1} = 0.02$, and $\epsilon_{c1} = 3$, $Q_{c1}(s)$ and $C_1(s)$ in (4.9) are given by:

$$Q_{c1}(s) = \frac{1.38(s^2+0.234s+0.117)(s^2+3.85s+2.02 \cdot 10^3)}{(s+1)(s+50)^3} \tag{4.68}$$

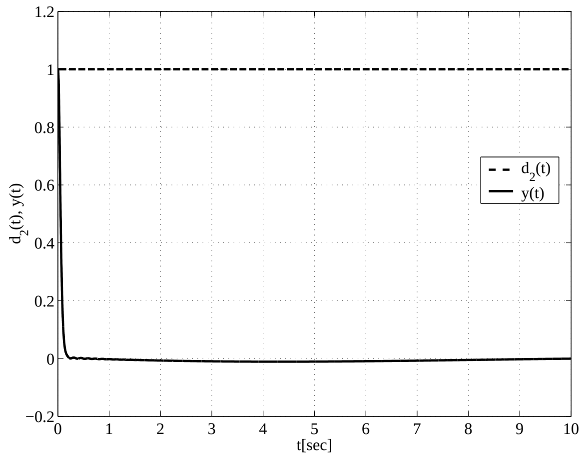


Fig. 4.3: Response of $y(t)$ with the one-degree-of-freedom control system for $d_2(t) = 1$

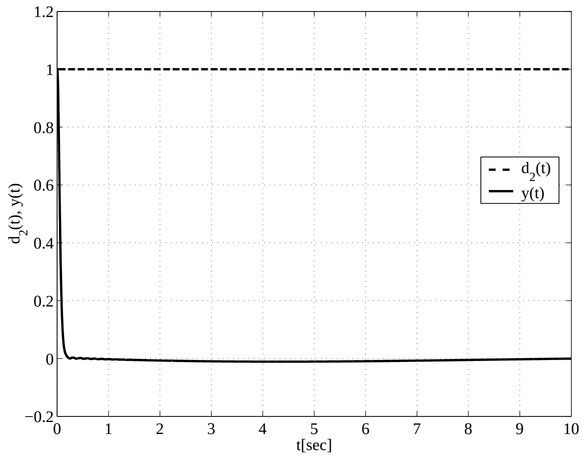


Fig. 4.4: Response of $y(t)$ with the two-degrees-of-freedom control system for $d_2(t) = 1$

and

$$C_1(s) = \frac{1.38(s^2 + 0.234s + 0.117)(s^2 + 3.85s + 2.02 \cdot 10^3)}{s(s + 0.167)(s^2 + 1.50 \cdot 10^2 s + 7.48 \cdot 10^3)}, \tag{4.69}$$

respectively.

Using the designed $C_1(s)$ in (4.69) and $C_2(s)$ in (4.66), the responses of the output $y(t)$ for step disturbance $d_2(t) = 1$ of the one-degree-of-freedom control system using $C_2(s)$ and two-degrees-of-freedom control system in Fig. 4.1 are shown in Fig. 4.3 and Fig. 4.4 , respectively. The solid line shows the response of the output $y(t)$ and the broken line shows that of the step disturbance $d_2(t) = 1$. Figure 4.3 and Fig. 4.4 show that the step disturbance $d_2(t) = 1$ is attenuated effectively. In addition, we find that the response of the two-degrees-of-freedom control system is the same as that of the one-degree-of-freedom control system.

On the other hand, the response of the output $y(t)$ for the step reference input $r(t) = 1$ of the one-degree-of-freedom control system using $C_2(s)$ and the two-degrees-of-freedom control system in Fig. 4.1 are shown in Fig. 4.5 and Fig. 4.6 , respectively. The solid line shows the

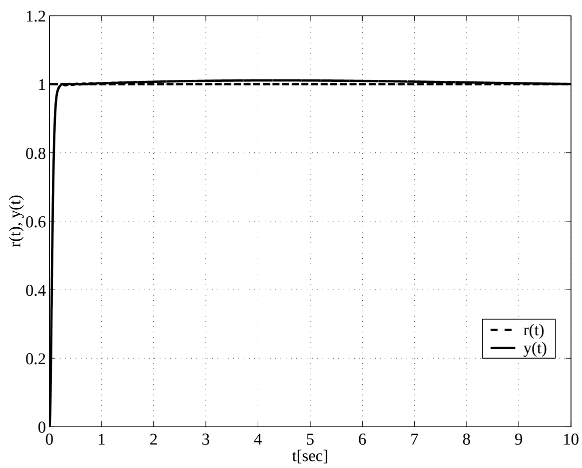


Fig. 4.5: Response of $y(t)$ for the one-degree-of-freedom control system for $r(t) = 1$

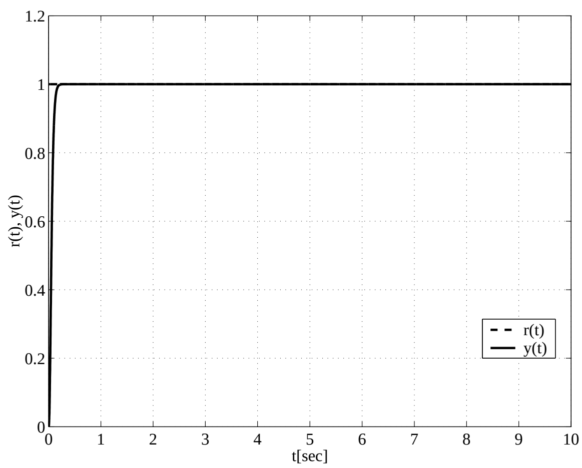


Fig. 4.6: Response of $y(t)$ for the two-degrees-of-freedom control system for $r(t) = 1$

response of the output $y(t)$ and the broken line shows that of the step reference input $r(t) = 1$.

Figure 4.5 and Fig. 4.6 show that these control systems are stable and the output $y(t)$ follows the step reference input $r(t) = 1$ without steady state error. In addition, to compare the responses of Fig. 4.5 and Fig. 4.6 , enlarged views from 0[sec] to 2[sec] are shown in Fig. 4.7 and Fig. 4.8 . Figure 4.7 and Fig. 4.8 show that the response of the two-degrees-of-freedom

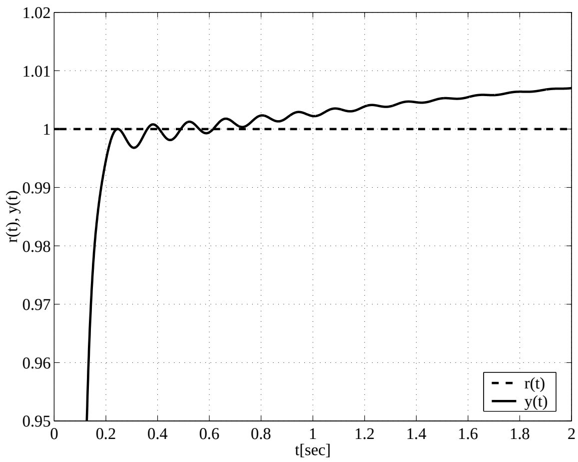


Fig. 4.7: Enlarged view from 0[sec] to 2[sec] of Fig. 4.5

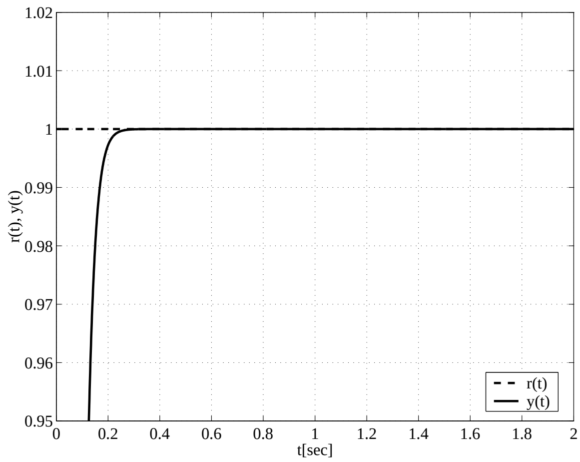


Fig. 4.8: Enlarged view from 0[sec] to 2[sec] of Fig. 4.6

control system has no overshoot and the settling time of the two-degrees-of-freedom control system is shorter than that of the one-degree-of-freedom control system.

We see that with the proposed two-degrees-of-freedom semistrongly stabilizing controller $C(s)$, the disturbance attenuation characteristic of the two-degrees-of-freedom control system can be the same as that of the one-degree-of-freedom control system and the input–output characteristic of the two-degrees-of-freedom control system can be different from that of the one-degree-of-freedom control system. That is, with the proposed controller, we can realize more accurate control for the reference input.

## 4.6 Conclusions

In this chapter, the parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers for semistrongly stabilizable plants was proposed. A design method for $Q_u(s) \in \mathcal{U}$ that satisfies (4.12) and makes $Q_{c2}(s)$ proper was then presented. Finally, a numerical example was presented to compare the responses of the one-degree-of-freedom control system [29] and the two-degrees-of-freedom control system to show the effectiveness of the proposed method.

In future work, two-degrees-of-freedom semistrongly stabilizing controllers for plants with time-delay will be considered.

# Chapter 5

# Conclusion

In this thesis, parameterizations on the semistrong stabilization were proposed.

In Chapter 2, we proposed the parameterization of all semistrongly stabilizable plants. A numerical example was presented to show that the plant in the proposed parameterization is surely semistrongly stabilizable.

In Chapter 3, we proposed the parameterization of all semistrongly stabilizing controllers for semistrongly stabilizable plants in Chapter 2. Control characteristics and a design method of the semistrongly stabilizing controller were also presented. A numerical example was illustrated to show that the controller designed by the presented design method surely works as semistrongly stabilizing controller. In this example, the angular velocity control of the two-inertia system was adopted in order to show the efficiency for the practical control.

In Chapter 4, we proposed the parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers for semistrongly stabilizable plants in Chapter 2. Control characteristics and a design method of the semistrongly stabilizing controller were also presented. A numerical example was illustrated to show the effectiveness for the proposed parameterization in Chapter 3 by comparison for responses of the numerical example in Chapter 3.

In future work, two-degrees-of-freedom semistrongly stabilizing controllers for plants with time-delay will be considered.

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# Publication papers

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Chapter 3 ○ Tatsuya Hoshikawa, Kou Yamada and Yuko Tatsumi, The parameterization of all semistrongly stabilizing controllers, *International Journal of Innovative Computing, Information and Control*, Vol.11, No.4, pp.1127-1137, 2015.

Chapter 4 ○ Tatsuya Hoshikawa, Kou Yamada and Yuko Tatsumi, The parameterization of all two-degrees-of-freedom semistrongly stabilizing controllers, *International Journal of Innovative Computing, Information and Control*, accepted for publications.