学 位 論 文 の 要 旨

円環板および円形境界を含む薄肉構造の非線形振動に関する研究
(Nonlinear Vibrations of Annular Plates and a Thin-Walled Structure with Circular Boundaries)

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近年，MEMS(Micro Electro Mechanical Systems)技術の発達により，センサやアクチュエータの小型化や軽量化が進んでいる．それに伴い，薄肉の円環板やはりと円板を結合した要素など円形境界を有する薄肉板がセンサやアクチュエータの構造要素に用いられる．薄肉の円環板はセンサのダイヤフラム等に用いられる．薄肉のはりと円板を結合した要素は MEMS スキャナに用いられる．薄肉板に横方向の周期外力が作用すると，大振幅共振応答によるたわみと面内変位の連成により非線形振動が発生する．さらに，特定の条件下ではカオス振動の発生も予想される．カオス振動は非周期的な応答であり加振振動数以外に様々な振動数成分を含む．このため，カオス振動はセンサやアクチュエータの信頼性や精度に影響を及ぼし得る．板の厚肉化による曲げ剛性の向上によりカオス振動への対策が可能である．しかし，機械要素の微細化に伴い，その対策が困難な場合もある．そこで，薄肉板の動的な振動挙動を予め考慮した設計，すなわち動的設計が必要となる．薄肉板の動的設計をするために，カオス振動に関する基礎資料が求められる．カオス振動は非周期的なため，時間変化に対する振動モードの変動が予想される．カオス振動が発生した際に薄肉板にどのような影響があるか評価するため，カオス振動に寄与する振動モードの時間変化の解明が工学上重要となる．

本研究では円環板ならびに，はりと円板を組み合わせた形状の結合要素について非線形振動に関する研究を行った．円環板では，外周自由・内周固定と外周固定・内周自由の境界条件を考えた．すべての試験片において実験を行い，外周自由と内周固定の円環板についてはあわせて解析も行った．

非線形振動実験では，まず試験片の基本特性として自重下での変形形状，線形固有振動数と固有振動モードならびに復元力特性を測定した．復元力特性は集中荷重に対する静たわみの関係から得た．ついで，試験片に横方向周期加振加速度を与え，非線形の周波数応答曲線を得た．加振振動数の掃引中は一定の加振加速度振幅とした．カオス振動応答を示す加振振動数において時系列波形とポアンカレ写像図を収録した．時系列波形の分析には

周波数分析，最大リャプノフ指数ならびに主成分分析を用いた．特に，カオス振動応答における振動モードの時間変化を調べるために，主成分分析に用いる時間長を短時間間隔に分割し，それぞれの区間について分析を行った．

外周自由と内周固定の円環板の解析では，固定境界において面内変位を仮定した．ハミルトンの原理から基礎式と適合条件式を得る．座標関数として半径方向はべき級数，周方向は正弦関数と余弦関数を用いた．適合条件式から応力関数を決定する．応力関数の同次解は解析的に決定される．同次解を用いて定数変化法により非同次解は決定される．面内方向の境界条件により同次解に含まれる未知数は決定される．得られたたわみと応力関数を基礎式に代入し，運動方程式を得る．ガラーキン法を用いて基礎式を有限多自由度系へ変換し，非線形連立常微分方程式を得る．たわみを静たわみと静的平衡位置を基準とした動的応答の和で表現する．静たわみについての非線形連立方程式から自重下での変形形状と復元力特性を得る．外力と非線形項を省略した動的応答についての線形連立常微分方程式から線形固有振動数と固有振動モードを得る．得られた固有振動モードに基づき非線形連立常微分方程式を基準座標へ変換する．Runge-Kutta-Gill 法による直接数値積分により板の応答を得た．

それぞれの試験片で発生したカオス振動で，振動モードの寄与率の不規則な時間変化を確認した．外周自由と内周固定の円環板では，複数の振動モードで寄与率の順番の不規則な入れ替わりを確認した．外周固定と内周自由の円環板では顕著に寄与する三つの振動モードにおいて，内部共振条件を満たしている二つの振動モードで寄与率の不規則なやり取りが確認された．一方，二次の高調波共振応答としてカオス振動に寄与する振動モードでは，ほぼ一定の寄与率を示した．はりと円板を組み合わせた結合要素に発生したカオス振動では，最低次曲げ振動モードとねじり振動モードの寄与が顕著である．最低次曲げ振動モードとねじり振動モードの間で寄与率の不規則なやり取りが示された．特徴的な寄与率の時間変化を示す三つの時間帯の存在を確認した．これらは，最低次曲げ振動モードの寄与率が支配的な時間帯，ねじり振動モードの寄与率が最低次曲げ振動モードより高くなる時間帯，最低次曲げ振動モードとねじり振動モードの寄与率が不規則に入れ替わる時間帯である．

外周自由・内周固定と外周固定・内周自由の円環板では，カオス振動において周方向へ不規則に変動する節直径を有する振動モードの存在を確認した．

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In recent years, sensors and actuators become small and light-weight due to development of MEMS (Micro Electro Mechanical System) technology. In the sensors and the actuators, thin plates such as annular plates and the combined structures with segments of beams and a disc are used as structural elements. The annular plates are used for diaphragms in sensors and the combined structures are used as elements of MEMS scanners. When the thin plate is subjected to periodic external force laterally, nonlinear vibrations are generated on the plate because deflection and in-plane displacement are coupled by large amplitude resonant responses. Furthermore, chaotic vibrations can be generated on the plate in the typical condition. The chaotic vibrations show non-periodic responses and contain broadband frequency components as well as excitation frequency. Hence, reliability and precision of the sensors and the actuators can be influenced by the chaotic vibrations. The chaotic vibrations can be avoided by an increase of thickness of the plate because the bending stiffness increases. However, it might be difficult in the MEMS because the dimension is restricted. Therefore, dynamic behavior of the plate has to be taken into consideration of the design. For the design, the fundamental information about the chaotic vibrations of the plate is required. In the chaotic vibrations, the vibration modes may fluctuate with respect to time because the chaotic vibrations are non-periodic. To evaluate the effect of the chaotic vibrations on the plate, it is important for engineering to investigate changes of the vibration modes with respect to time.

In this research, nonlinear vibrations of the annular plates and the combined structures with segments of beams and a disc are investigated. Two kinds of annular plates are examined; An outer-free and inner-clamped annular plate and an outer-clamped and inner-free annular plate. Experiments of the nonlinear vibrations are conducted on the annular plates and the combined structure. Analytical results are also shown about the nonlinear vibrations of the annular plate with outer-free and inner-clamped edges.

In the experiment, configuration of the plate under the gravity, linear natural frequencies and corresponding modes of vibration, characteristics of the restoring force of the plate are measured as the fundamental properties of the plate. As the characteristics of the restoring force of the plate, relation between static deflection and concentrated force is measured. Then, applying periodic acceleration to the plate, nonlinear frequency response curves are obtained. The amplitude of the periodic acceleration is kept constant during sweeping the excitation frequencies. Time histories and the Poincare projection of the chaotic responses are measured in the typical frequency region. The time histories are examined by the Fourier spectra, the maximum Lyapunov exponents and the principal component analysis. To investigate the fluctuation of the vibration modes with respect to time, the principal component analysis is adapted on the time histories, which are divided into short time intervals.

In the analysis of the outer-free and inner-clamped annular plate, the in-plane displacement at the clamped edge is assumed. The governing equation and the compatibility equation are obtained with the Hamilton's principle. The coordinate function of the deflection in the radial direction is introduced as the power series while that in the circumferential direction is the sine and cosine functions. The homogeneous solutions can be exactly obtained. The particular solutions of the stress function are obtained with the variation of constants method. Unknown constants in the homogeneous solution are determined in terms of deflection and parameters, with which the in-plane boundary condition is exactly satisfied. Substituting the deflection and the stress function into the governing equation, the equation of motion is obtained. With the Galerkin procedure, the equation is reduced to a set of nonlinear ordinary differential equations. The deflection is assumed to be the sum of static deflection and dynamic deflection. The deformed configuration under the gravity and the restoring force of the plate is obtained from a set of simultaneous equation in terms of the static deflection. The natural frequencies and corresponding vibration modes are obtained from linear ordinary differential equation in terms of dynamic deflection. With the vibration modes, the nonlinear ordinary differential equation in terms of the dynamic deflection is transformed into that in terms of the normal coordinate. The responses of the plate are calculated by numerical integration with the Runge-Kutta-Gill method.

As a result, fluctuation of the contribution ratio with respect to time is shown in the chaotic responses in each plate. In the outer-free and inner-clamped annular plate, it is confirmed that the order of the contribution ratio of multiple modes of vibration is irregularly changed. In the outer-clamped and inner-free annular plate, three modes of vibration predominantly contribute to the chaotic response. The two modes of vibration, which satisfy with the condition of internal resonance, show the exchange of the contribution ratio. On the other hand, the mode of vibration, which contributes to the chaotic response as the super-harmonic resonance of second order, shows almost constant contribution ratio. In the combined structure, the lowest flexural mode and the torsional mode contribute to the chaotic response predominantly. Irregular change of the contribution ratio between the lowest flexural mode and the torsional mode is shown. The change of the contribution ratio can be classified into three kinds of time periods; The time period with the dominant contribution of the lowest flexural mode, the time period with larger contribution ratio of the torsional mode than that of the lowest flexural mode and the time period with irregular change of the order of the contribution ratio between the lowest flexural mode and the torsional mode.

Furthermore, irregular variation of the vibration modes in the circumferential direction is observed in the annular plates.