令和２年度　博士論文

Observer design for control systems

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Chapter 1

Introduction

1.1 State Observer

A state of a plant in control system should be able to determine and measure in order to design the appropriate system along with the controlled input and the expected output. All factors in the controlled plant could be theoretically measured; however, some variables of the plant were not able to measure due to either missing the values from controlled input or incorrect output such as from sensors. This is the reason that the state of the plant is often invalid. In order to design a control system for the plant which considering such not all state variables can be measured, the estimation called a state observer has been employed to validate the unavailable state variable of the controlled plant. The state observer theory is initially established by Luenberger [1, 2, 3]. Subsequently, more design methods to obtain the state variables have been derived as the parametrization of all state observers [4] and of all linear functional observers [5] are derived. Those works require accessing to the control input for estimation [1, 2, 3, 4, 5]

As mentioned previously, some case such as estimation of an IC engine torque or velocity and angle of planar gantry crane, both the state variable and the control input of the system are unavailable [6, 7]. Thus, the state observer for a plant not to access to the control input is required to estimate the state variable only using only the measured output. such a state observer is called the unknown input observer. Only the measured output could be obtained from the system and used to estimate the state of such plant. Using the state observer for a plant which not to access to the control input to estimate the state variable, such a state observer is called the unknown input observer. The unknown input observer firstly is pursued by Kudva, Viswanadham and Ramakrishna [8]. Since then, several papers about designing unknown input observers have been published [9, 10, 11]. According to these papers, the unknown input observer for the plant $(A, B, C, 0)$ can be designed if and only if the following two conditions hold to be true in cases that: (c1) $\operatorname{rank} CB = \operatorname{rank} B$ and (c2) the plant $(A, B, C, 0)$ has no invariant zero in the closed right half plane. The first condition (c1) implies that the number of outputs should be greater than or equal to that of inputs. The second condition (c2) means that the controlled plant is of non-minimum phase. Since the conditions (c1) and (c2) are rather restrictive, a number of authors have considered designing unknown input observers by relaxing the the conditions (c1) and (c2). Some works have considered the problem of designing unknown input observers without requiring the first condition [12, 13, 14, 15] considered the problem of designing unknown input observers without requiring the first condition (c1). Unfortunately, the design methods in [12, 13, 14, 15] Unfortunately, those design methods as mentioned cannot be applied if the second condition (c2) fails. (c2) fails. In contrast, Hikita [16] and K. Fuwa, T. Narikiyo, et al [16, 17] has tackled approximately lifting both of the first and second conditions (c1)-(c2) via the minimal polynomial bases approach and eigenstructure assignment approach, respectively. Those approaches are highly algebraic and it is difficult to intuitively tune the input-output characteristics of the resulting control systems A. Termehchy and A. Afshar [24] proposes to augment the controlled plant with a low-pass filter so that the augmented controlled plant satisfies the conditions (c1)-(c2). Their design method requires to increase the number of sensors for measuring the overall output of the augmented plant, and hence from a cost-aware point of view it is not readily employed when the original plant is given. The theory of unknown input observer is also applied to the systems with non-linearities or time-varying parameters. [24] requires to increase the number of sensors for measuring the overall output of the augmented plant, and hence from a cost-aware point of view it is not readily employed when the original plant is given. The unknown input observer is also applied to the systems with non-linearities or time-varying parameters [25, 26].

1.2 Disturbance Observer.

The state observer can also be used to estimate the disturbances in the system. Such an observer is called a disturbance observer. Disturbance observers are used in the motion-control field to predict disturbances and make a closed loop system to be robustly stable [28, 29, 30, 31, 32, 33, 34, 35, 36]. Generally, the disturbance observer both includes the disturbance signal generator and observer. Usually the disturbance is assumed to estimate as a step disturbance, because the disturbance observer has a simple structure and is easy to used in many applications[28, 29, 30, 31, 32, 33, 34, 35, 36].

Disturbance observer-based control has been seen as the most promising approach to attenuate disturbances [37]. To linear systems , Li et al. proposed the frequency domain disturbance observer, time domain disturbance observer and extended state observer in different domain for linear systems. For the nonlinear systems, Li et al. and Chen et al [37, 38]. proposed nonlinear disturbance observer for constant disturbances and nonlinear disturbance observer for general exogenous disturbances based on different disturbances. Nevertheless, there is not available for parameterization of all disturbance observers for any disturbance. For solving this problem, Yamada et al. proposed a parameterization of linear disturbance observer for specific constant disturbances by applying the inputs and outputs of system as variables[39]. However, not all parameterizations of linear disturbance observer has been obtained, it is still a disturbance observer for specific disturbances. In addition, the variables of disturbance observer in [39] Due to the diversity of systems, there are still lots of work to be solved about the parameterization of other various systems.

1.3 The purpose and contents of this study

In this thesis, we propose a design method of unknown input observer for non-minimum phase plants and a parameterization of all linear disturbance observers using the states and inputs of the system as variables for constant disturbances.

In chapter 2, we propose alternative design methods of unknown input observers for non-minimum phase plants that are handily applicable when the intended bandwidth of the control system is specified. The proposed design methods do not require neither of the conditions (c1)-(c2) nor plant augmentation [24]. In this proposed method in chapter 2, if we design a state observer discarding the high-frequency-range signal components of the control input, then the resulting state observer works as an unknown input observer. Furthermore, as a complement to the proposed design methods, we describe that the resulting unknown input observers can be employed for constructing output feedback control systems if it is combined with the H_∞ state feedback control [22].

In chapter 3, we propose the parameterization of linear disturbance observers for constant disturbances. By using the states and inputs of the system as variables, the parameterization for constant disturbances was obtained.

In Chapter 4 Summaries the result of the present study by the conclusion.

Notations

R	the set of real numbers.
$\mathcal{L}(x(t))$	the laplace transformation of $x(t)$.
$R(s)$	the set of real rational functions with s.
RH_∞	the set of stable proper real coefficient rational functions.
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	represents the state space description $C(Is-A)^{-1}B+C$

Chapter 2

A design method of unknown input observers

2.1 introduction

A state observers are employed to estimate the unavailable state variable of the controlled plant. The state observer theory is initially established by Luenberger [1, 2, 3]. Subsequently, the parametrization of all state observers [4] and of all linear functional observers [5] are derived. The design methods in [1, 2, 3, 4, 5] require the access to the control input for estimating the state variable. In contrast, in this chapter, we address design methods of unknown input observers, which are state observers independent of the control input.

In some cases such as estimation of an IC engine torque [6] and velocity and angle of planar gantry crane [7], both the state variable and the control input are unavailable, and state observers is required to estimate the state variable only using only the measured output. Such a state observer is called the unknown input observer. That is, the unknown input observer has been used to estimate the state variable of the plant in the presence of unknown input. In addition, the unknown input observer is applied to the systems with nonlinearities or time-varying parameters [25, 26]. Initially, the unknown input observer is examined by Kudva, Viswanadham and Ramakrishna [8]. Since then, several papers have been published to design unknown input observers [9, 10, 11]. According to these papers, the unknown input observer for the plant $(A, B, C, 0)$ can be designed if and only if the following two conditions hold true: (c1) rank $CB =$ rank B and (c2) the plant $(A, B, C, 0)$ has no invariant zero in the closed right half plane. The first condition (c1) implies that the number of outputs should be greater than or equal to that of inputs. The second condition (c2) means that the controlled plant is of non-minimum phase. Since the conditions (c1) and (c2) are rather restrictive, a number of authors have considered designing unknown input observers by relaxing the the conditions (c1) and (c2). The papers [12, 13, 14, 15] considered the problem of designing unknown input observers without requiring the first condition (c1). Unfortunately, the design methods in [12, 13, 14, 15] cannot be applied if the second condition (c2) fails. In contrast, the papers [16, 17] tackled approximately lifting both of the first and second conditions (c1)-(c2) via the minimal polynomial bases approach and eigenstructure assignment approach, respectively. Those approaches are highly algebraic and it is difficult to intuitively tune the input-output characteristics of the resulting control systems. The paper [24] proposes to augment the controlled plant with a low-pass filter so that the augmented controlled plant satisfies the conditions (c1)-(c2). The design method in [24] requires to increase the number of sensors for measuring the overall output of the augmented plant, and hence from a cost-aware point of view it is not readily employed when the original plant is given.

In this chapter, we propose alternative design methods of unknown input observers for non-minimum phase plants, such that are handily applicable when the intended bandwidth of the control system is specified. The proposed design methods do not require neither of the conditions (c1)-(c2) nor plant augmentation [24]. The idea behind that is as follows: If the controlled plant is intended to function in the lower-frequency range, the control input mainly contains low-frequency-range signal components, that is, the control input decays in the higher frequency range. Noting that, if we design a state observer discarding the high-frequency-range signal components of the control input, then the resulting state observer works as an unknown input observer. In order to embody this idea, we utilize the parametrization of all state observers [4, 5] and stable left filtered inverses [20, 21] as underlying techniques. It is shown that the stable left filtered inverses [20, 21] enable to determine the Youla parameter in the state observer parametrization so that the resulting observer works as an unknown input observer. Furthermore, as a complement to the proposed design methods, we describe that the resulting unknown input observers can be employed for constructing output feedback control systems if it is combined with the H_∞ state feedback control [22].

This chapter is organized as follows: In Section 2.2, the problem considered in this chapter is formulated. In Section 2.3, we derive the unknown input observer design methods. In Section 2.4, we construct an output feedback control by employing the proposed unknown input observer. In Section 2.5, the features of the resulting control systems are illustrated through numerical examples. In Section 2.6, the present contributions are summarized.

2.2 Problem formulation

Consider the plant written by

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + Bu(t) \\ y(t) & = & Cx(t) \end{array} \right. , \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the state variable, $u(t) \in \mathbb{R}^p$ is the control input, $y(t) \in \mathbb{R}^m$ is the measured output, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times p}$, $C \in \mathbb{R}^{m\times n}$. The transfer function from the control input $u(s)$ to the measured output $y(s)$ in (2.1) is denoted by

$$y(s) = G(s)u(s), \tag{2.2}$$

where

$$G(s) = C\left(sI - A\right)^{-1} B \in R^{m\times p}(s), \tag{2.3}$$

where $R(s)$ denotes the set of real-rational transfer functions. It is assumed that (A, B) is stabilizable, (C, A) is detectable, and $G(s)$ is of full row normal rank:

$$\text{rank } G(s) = p, \tag{2.4}$$

and has no invariant zero on the imaginary axis, Furthermore, this chapter focuses on the situation that the frequency component range of the control input $u(j\omega)$ is limited to $0 \leq \omega \leq \omega_{max}$, where ω_{max} specifies the maximum frequency component of the control input $u(j\omega)$. We note that $G(s)$ is allowed to have some invariant zeros in the open right half plane, that is, $G(s)$ is allowed to be of non-minimum phase.

When the state variable $x(t)$ in (2.1) is not available, as is the case in many practical control problems, we employ state observers, which estimate the state variable $x(t)$ in (2.1) utilizing the available information on $y(t)$ and $u(t)$. The general form of state estimates based on the available information on $y(t)$ and $u(t)$ is given as follows (Fig. I):

$$\xi(s) = F_1(s)y(s) + F_2(s)u(s), \tag{2.5}$$

where $\xi(t) \in \mathbb{R}^n$ is the estimate of the state variable $x(t)$. The transfer functions $F_1(s) \in R^{n\times m}(s)$ and

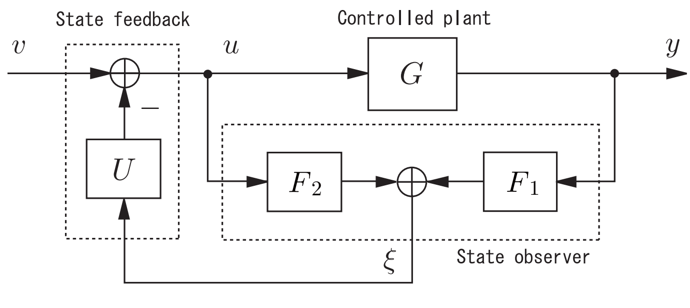

Fig. I: Configuration of control system.

$F_2(s) \in R^{n\times p}(s)$ in (2.5) are required to satisfy the condition

$$\lim_{t\to\infty} (x(t) - \xi(t)) = 0. \tag{2.6}$$

In some cases, not only the state variable $x(t)$ but also the control input $u(t)$ is unavailable and the state observer (2.5) depend only on the measured output $y(t)$. Such a state observer is called an unknown input (state) observer. The purpose of this chapter is to propose alternative design methods of unknown input observers for the non-minimum phase plant (2.1).

2.3 Design methods of unknown input observers

In this section, we describe the unknown input observer design methods. According to [4, 5], the parametrization of all state observer in (2.5) for the plant $G(s)$ in (2.1) is written by

$$F_1(s) = (sI - A + BU)^{-1} BX(s) + Q(s)\tilde{D}(s) \tag{2.7}$$

and

$$F_2(s) = (sI - A + BU)^{-1} BY(s) - Q(s)\tilde{N}(s), \tag{2.8}$$

where $U \in \mathbb{R}^{p\times n}$ makes $A - BU$ have no eigenvalue in the closed right half plane. Furthermore, $\tilde{N}(s) \in RH_\infty^{m\times p}$ and $\tilde{D}(s) \in RH_\infty^{m\times m}$ are coprime factors of $G(s)$ on RH_∞ (i.e. the set of stable real-rational functions) satisfying

$$G(s) = \tilde{D}^{-1}(s)\tilde{N}(s) = N(s)D^{-1}(s), \tag{2.9}$$

where $X(s) \in RH_\infty^{p\times m}$ and $Y(s) \in RH_\infty^{p\times p}$ are functions satisfying

$$\begin{aligned}\begin{bmatrix} Y(s) & X(s) \\ -\tilde{N}(s) & \tilde{D}(s) \end{bmatrix}\begin{bmatrix} D(s) & -\tilde{X}(s) \\ N(s) & \tilde{Y}(s) \end{bmatrix} &= \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \\ &= \begin{bmatrix} D(s) & -\tilde{X}(s) \\ N(s) & \tilde{Y}(s) \end{bmatrix}\begin{bmatrix} Y(s) & X(s) \\ -\tilde{N}(s) & \tilde{D}(s) \end{bmatrix}\end{aligned} \tag{2.10}$$

and $Q(s)$ is an arbitrary function in $RH_\infty^{n\times m}$.

From (2.8), it is observed that if there exists $Q(s) \in RH_\infty^{n\times m}$ satisfying

$$(sI - A + BU)^{-1}BY(s) - Q(s)\tilde{N}(s) = 0, \tag{2.11}$$

we can obtain an unknown input observer. However, in order to find $Q(s) \in RH_\infty^{n\times m}$ satisfying (2.11), $G(s)$ must be of minimum-phase.

In order to design unknown input observers for non-minimum phase plants, we adopt the following idea. If we design $Q(s) \in RH_\infty^{n\times m}$ such that

$$Q(j\omega)\tilde{N}(j\omega) \simeq (j\omega I - A + BU)^{-1}BY(j\omega) \ \ (0 \le \forall\omega \le \omega_{max}), \tag{2.12}$$

then

$$F_2(j\omega) \simeq 0 \ \ (0 \le \forall\omega \le \omega_{max}) \tag{2.13}$$

holds true. Together with (2.13), the assumption that the frequency component range of the control input $u(t)$ is limited to $0 \le \omega \le \omega_{max}$ implies

$$\bar{u}(t) = \mathcal{L}^{-1}\left\{F_2(s)u(s)\right\} \simeq 0, \tag{2.14}$$

where $\mathcal{L}^{-1}\{\cdot\}$ denotes the inverse Laplace transformation. Therefore, when $Q(s)$ is settled to satisfy (2.12), the state estimate $\xi(s)$ in (2.5) reduces to

$$\xi(s) = F_1(s)y(s) \tag{2.15}$$

with $F_1(s)$ defined by (2.7) working as an unknown input observer.

Hence in the rest of this section, we consider designing $Q(s) \in RH_\infty^{n\times m}$ which satisfies (2.12). Specifically, we propose to settle $Q(s)$ so that the following condition is satisfied:

$$Q(s)\tilde{N}(s) = (sI - A + BU)^{-1}BY(s)G_K(s)Q_l(s), \tag{2.16}$$

where $G_K(s) \in RH_\infty^{p\times p}$ is an inner part of the transfer function $\tilde{N}(s)$ with $G_K(0) = I$. Furthermore, $Q_l(s)$ is given as the diagonal matrix with $\dfrac{1}{(1+sT_1)^{\alpha_1}}$ on its i-th diagonal entry:

$$Q_l(s) = \text{diag}\left\{ \frac{1}{(1+sT_1)^{\alpha_1}} \ \ \cdots \ \ \frac{1}{(1+sT_p)^{\alpha_p}} \right\}, \tag{2.17}$$

where α_i $(i = 1,\dots,p)$ are positive integers chosen to make $Q(s)$ proper and T_i $(i = 1,\dots,p)$ are positive real numbers chosen to satisfy the condition

$$I - G_K(j\omega)\text{diag}\left\{ \frac{1}{(1+j\omega T_1)^{\alpha_1}} \ \ \cdots \ \ \frac{1}{(1+j\omega T_p)^{\alpha_p}} \right\} \simeq 0 \ \ (0 \le \forall\omega \le \omega_{max}). \tag{2.18}$$

The following identity confirms that $Q(s)$ settled by (2.16) satisfies (2.12):

$$\begin{aligned}&(j\omega I - A + BU)^{-1}BY(j\omega) - Q(j\omega)\tilde{N}(j\omega)\\ &= (j\omega I - A + BU)^{-1}BY(j\omega)\\ &\quad \left[I - G_K(j\omega)\mathrm{diag}\left\{\begin{array}{ccc}\frac{1}{(1+j\omega T_1)^{\alpha_1}} & \cdots & \frac{1}{(1+j\omega T_p)^{\alpha_p}}\end{array}\right\}\right]. \end{aligned} \tag{2.19}$$

Next, we provide state-space design methods of $Q(s) \in RH_\infty^{n\times m}$ satisfying (2.16). Before proceeding, let the state space realization of $\tilde{N}(s)$ in (2.10) be given by

$$\tilde{N}(s) = \left[\begin{array}{c|c}\tilde{A} & \tilde{B}\\ \hline \tilde{C} & 0\end{array}\right], \tag{2.20}$$

where generally speaking, $\left[\begin{array}{c|c}A & B\\ \hline C & D\end{array}\right]$ represents the transfer function $C(sI-A)^{-1}B+D$. Then we assume that it holds that

$$\mathrm{rank}\,\Phi = p, \tag{2.21}$$

where the matrix Φ is constructed from the parameters of the state-space realization of $\tilde{N}(s)$ as follows:

$$\Phi = \left[\begin{array}{c}\tilde{B}_1^{\mathrm{T}}\left(\tilde{A}^{\mathrm{T}}\right)^{\alpha_1-1}\tilde{C}^{\mathrm{T}}\\ \vdots\\ \tilde{B}_p^{\mathrm{T}}\left(\tilde{A}^{\mathrm{T}}\right)^{\alpha_p-1}\tilde{C}^{\mathrm{T}}\end{array}\right], \tag{2.22}$$

$$\tilde{B} = \left[\begin{array}{ccc}\tilde{B}_1 & \cdots & \tilde{B}_p\end{array}\right] \quad \left(\tilde{B}_i \in \mathbb{R}^n\ (i=1,\ldots,p)\right) \tag{2.23}$$

and

$$\alpha_i = \min\left(j|\tilde{B}_i^{\mathrm{T}}\left(\tilde{A}^{\mathrm{T}}\right)^{j-1}\tilde{C}^{\mathrm{T}} \neq 0; j=1,\ldots,n\right) \quad (i=1,\ldots,p). \tag{2.24}$$

We note that the assumption (2.21) means that $G(s)$ can be decoupled using static feedback control, and hence does not impose severe restriction. Under the assumption (2.21), we below propose (Method 1) and (Method 2) to determine $Q(s) \in RH_\infty^{n\times m}$ satisfying (2.16).

(Method 1) This method is based on the result in [20] and determines $Q(s) \in RH_\infty^{n\times m}$ as follows:

$$Q(s) = (sI - A + BU)^{-1}BY(s)\hat{G}(s), \tag{2.25}$$

where

$$\hat{G}(s) = \left[\begin{array}{c|c}\tilde{A} + K\bar{D}_l^{-1}X\hat{\Phi}^{\mathrm{T}}\tilde{C} & K\bar{D}_l^{-1}X\hat{\Phi}^{\mathrm{T}}\\ \hline \Gamma^{-1}\left(E^{-\frac{1}{2}}\right)^{\mathrm{T}}\bar{D}_l^{-1}X\hat{\Phi}^{\mathrm{T}}\tilde{C} & \Gamma^{-1}\left(E^{-\frac{1}{2}}\right)^{\mathrm{T}}\bar{D}_l^{-1}X\hat{\Phi}^{\mathrm{T}}\end{array}\right], \tag{2.26}$$

$$\Phi\hat{\Phi} = I_p, \tag{2.27}$$

$$X = \mathrm{diag}\left\{\begin{array}{ccc}\beta_{1\alpha_1} & \cdots & \beta_{p\alpha_p}\end{array}\right\}, \tag{2.28}$$

$$\beta_{ij} = \alpha_i C_j\left(T_i\right)^{-j} \quad (i=1,\ldots,p : j=1,\ldots,\alpha_i), \tag{2.29}$$

and $\bar{D}_l \in \mathbb{R}^{p\times p}$ is an arbitrary constant nonsingular matrix satisfying

$$E = \bar{D}_l^{-1}\left(\bar{D}_l^{-1}\right)^{\mathrm{T}}. \tag{2.30}$$

Furthermore, the auxiliary feedback gain K and scaling matrix Γ are defined by

$$K = -\Psi X^{-1}\bar{D}_l - P\left(\bar{D}_l^{-1}X\hat{\Phi}^{\mathrm{T}}\tilde{C}\right)^{\mathrm{T}}\left(E^{-1}\right)^{\mathrm{T}}, \tag{2.31}$$

$$\Gamma = -\left(E^{-\frac{1}{2}}\right)^{\mathrm{T}}\bar{D}_l^{-1}X\hat{\Phi}^{\mathrm{T}}\tilde{C}\left(\tilde{A} + K\bar{D}_l^{-1}X\hat{\Phi}^{\mathrm{T}}\tilde{C}\right)^{-1}\left(\Psi X^{-1} + K\bar{D}_l^{-1}\right) + \left(E^{-\frac{1}{2}}\right)^{\mathrm{T}}\bar{D}_l^{-1}, \tag{2.32}$$

$$\Psi = \left[\begin{array}{ccc}\tilde{A}^{\alpha_1}\tilde{B}_1 + \cdots + \beta_{1\alpha_1}\tilde{B}_1 & \ldots & \tilde{A}^{\alpha_p}\tilde{B}_p + \cdots + \beta_{p\alpha_p}\tilde{B}_p\end{array}\right], \tag{2.33}$$

where $P = P^{\mathrm{T}} \geq 0$ is the unique solution of the Riccati equation

$$P\left(\tilde{A}^{\mathrm{T}} - \tilde{C}^{\mathrm{T}}\hat{\Phi}\Psi^{\mathrm{T}}\right) + \left(\tilde{A}^{\mathrm{T}} - \tilde{C}^{\mathrm{T}}\hat{\Phi}\Psi^{\mathrm{T}}\right)^{\mathrm{T}} P - P\left(\bar{D}_l^{-1}X\hat{\Phi}^{\mathrm{T}}\tilde{C}\right)^{\mathrm{T}} E^{-1}\left(\bar{D}_l^{-1}X\hat{\Phi}^{\mathrm{T}}\tilde{C}\right) P = 0 \tag{2.34}$$

to make $\tilde{A} + K\bar{D}_l^{-1}X\hat{\Phi}^{\mathrm{T}}\tilde{C}$ have no eigenvalue in the closed right half plane.

(Method 2) This method is based on the result in [21] and determines $Q(s) \in RH_\infty^{n\times m}$ as follows:

$$Q(s) = (sI - A + BU)^{-1}BY(s)G_K(s)G_0(s), \tag{2.35}$$

where

$$G_0(s) = \left[\begin{array}{c|c} \tilde{A} - \Psi\hat{\Phi}^{\mathrm{T}}\tilde{C} & -\Psi\hat{\Phi}^{\mathrm{T}} \\ \hline X\hat{\Phi}^{\mathrm{T}}\tilde{C} & X\hat{\Phi}^{\mathrm{T}} \end{array}\right] = \begin{bmatrix} G_{01}(s) \\ \vdots \\ G_{0p}(s) \end{bmatrix} \quad \left(G_{0i}(s) \in R^{1\times m}(s)(i = 1, \ldots, p)\right), \tag{2.36}$$

$\hat{\Phi}$, X, β_{ij} and Ψ are given by (2.27), (2.28), (2.29) and (2.33), respectively. In addition, $G_K(s)$ is designed as follows: Let the minimal realization of $G_{0i}(s)\ (i = 1, \ldots, p)$ be

$$G_{0i}(s) = \left[\begin{array}{c|c} A_{0i} & B_{0i} \\ \hline C_{0i} & D_{0i} \end{array}\right] \quad (i = 1, \ldots, p). \tag{2.37}$$

Then, from this realization, $G_K(s)$ is obtained by

$$\begin{aligned} G_K(s) &= \mathrm{diag}\left\{ \frac{1}{1 + C_{01}\left(sI - A_{01}\right)^{-1}K_1} \quad \cdots \quad \frac{1}{1 + C_{0p}\left(sI - A_{0p}\right)^{-1}K_p} \right\} \\ &= \left[\begin{array}{cc|cc} A_{01} - K_1C_{01} & 0 & K_1 & 0 \\ & \ddots & & \ddots \\ 0 & A_{0p} - K_pC_{0p} & 0 & K_p \\ \hline -C_{01} & 0 & 1 & 0 \\ & \ddots & & \ddots \\ 0 & -C_{0p} & 0 & 1 \end{array}\right], \end{aligned} \tag{2.38}$$

where

$$K_i = P_iC_{0i}^{\mathrm{T}}(i = 1, \ldots, p) \tag{2.39}$$

and $P_i \geq 0(i = 1, \ldots, p)$ is the unique stabilizing solution of the Riccati equation

$$P_iA_{0i}^{\mathrm{T}} + A_{0i}P_i - P_iC_{0i}^{\mathrm{T}}C_{0i}P_i = 0\ (i = 1, \ldots, p). \tag{2.40}$$

The key point common in (Method 1) and (Method 2) is that the Youla parameter $Q(s)$ includes a stable left filtered inverse of $\tilde{N}(s)$. In (Method 1), $\hat{G}(s)$ is the stable left filtered inverse, and yields the inner function $G_K(s)$ for (2.16), which is not necessarily diagonal. In (Method 2), $G_K(s)G_0(s)$ is the stable left filtered inverse, and yields the inner function $G_K(s)$ for (2.16), which has the diagonal structure.

2.4 Output feedback controller design

In accordance with the proposed unknown input observer design methods, this section describes how to construct the output feedback control system in Fig. I.

Consider the output feedback control

$$u(t) = -U\xi(t) + v(t) \tag{2.41}$$

for the controlled plant $G(s)$, where $\xi(t)$ is the state estimate of the state variable $x(t)$ and $v(t) \in \mathbb{R}^p$ is an external input exerted on the control system. A method of designing the state feedback gain U and state estimate $\xi(t)$ for the output feedback control (2.41) is summarized as follows:

1. Specify the frequency component range $0 \leq \omega \leq \omega_{max}$ from the supposed bandwidth of the external input $v(t)$.

2. Using the design method of H_∞ state feedback controllers in [22], fix U in (2.41) so that the maximal singular value of the transfer function from $v(j\omega)$ to $u(j\omega)$ is made negligible outside the frequency component range $0 \leq \omega \leq \omega_{max}$.

3. Using (Method 1) or (Method 2) in Section 2.3, design the unknown input observer (2.5) which produces the state estimate $\xi(t)$ used for (2.41).

2.5 Numerical example

In this section, we design the output feedback control in Section 2.4 for two sample cases, and examine the features of the proposed unknown input observer design methods.

2.5.1 Numerical example 1

Consider employing (Method 1) in Section 2.3 to design the output feedback control (2.41) for the controlled plant $G(s)$ written by

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & \begin{bmatrix} -10 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & 0 & -30 & 0 \\ 0 & 0 & 0 & -30 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u(t) \\ y(t) & = & \begin{bmatrix} 2 & 0 & 4 & 0 \\ 0 & 4 & 0 & 5 \end{bmatrix} x(t) \end{array} \right. . \tag{2.42}$$

The above controlled plant is of non-minimum phase, since it has invariant zeros at $(10, 0)$ and $(20, 0)$.

It is supposed that the external input $v(t)$ in (2.41) and initial state $x(0)$ are given by

$$v(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} \sin(0.1t) \\ 2\sin(0.1t) \end{bmatrix} \tag{2.43}$$

and

$$x(0) = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}^{\mathrm{T}}, \tag{2.44}$$

respectively. Referring to the angular frequency of the external input $v(t)$, we specify the frequency component range by $\omega_{max} = 0.1$.

Using the method in [18], $\tilde{N}(s)$ satisfying (2.9) is obtained as

$$\tilde{N}(s) = \left[\begin{array}{cccc|cc} -10 & 0 & 0 & 0 & 1 & 0 \\ 0 & -20 & 0 & 0 & 0 & 1 \\ 0 & 0 & -30 & 0 & 1 & 0 \\ 0 & 0 & 0 & -30 & 0 & 1 \\ \hline 2 & 0 & 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 5 & 0 & 0 \end{array} \right]. \tag{2.45}$$

The matrix Φ in (2.22), constructed from the state-space representation (2.45), satisfies the condition

$$\text{rank}\ \Phi = 2, \tag{2.46}$$

as Φ in (2.22) is given by

$$\Phi = \begin{bmatrix} 6 & 0 \\ 0 & 9 \end{bmatrix}, \tag{2.47}$$

with $\alpha_1 = 1$ and $\alpha_2 = 1$. By (2.46), $\hat{\Phi}$ satisfying (2.27) is obtained as

$$\hat{\Phi} = \begin{bmatrix} 0.1667 & 0 \\ 0 & 0.111 \end{bmatrix}. \tag{2.48}$$

We choose the time constants in (2.17) as $T_1 = 0.001$, $T_2 = 0.002$ so that the condition (2.18) is satisfied in the frequency component range $0 \le \omega \le \omega_{max} = 0.1$. Setting $\bar{D}_l = I$, together with (2.28), (2.29), (2.30), (2.31), (2.33) and (2.34), we have

$$\left\{ \begin{array}{rcl} \beta_{11} & = & 1000 \\ \beta_{21} & = & 500 \end{array} \right. , \tag{2.49}$$

$$X = \begin{bmatrix} 1000 & 0 \\ 0 & 500 \end{bmatrix}, \tag{2.50}$$

$$E = I, \tag{2.51}$$

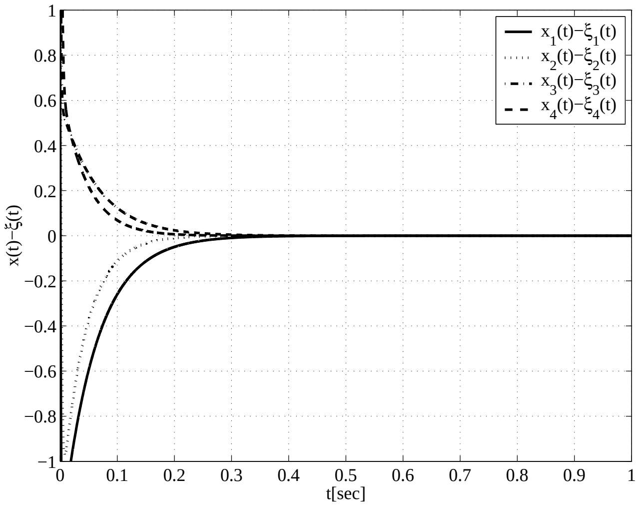

Fig. II: Time response of state estimation error $x(t) - \xi(t)$.

$$\Psi = \begin{bmatrix} 990 & 0 \\ 0 & 480 \\ 970 & 0 \\ 0 & 470 \end{bmatrix}, \tag{2.52}$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.53}$$

and

$$K = \begin{bmatrix} -0.990 & 0 \\ 0 & -0.960 \\ -0.970 & 0 \\ 0 & -0.940 \end{bmatrix}. \tag{2.54}$$

Substituting the above parameters into (2.25), $Q(s)$ is obtained. Consequently, the unknown input observer (2.5) reduces to (2.15) in the intended bandwidth of the control system.

The state estimation error

$$e(t) = x(t) - \xi(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} - \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \\ \xi_3(t) \\ \xi_4(t) \end{bmatrix} \tag{2.55}$$

evolves over time as depicted in Fig. II, where the solid, dotted, alternate long/short dash, broken lines correspond with $x_1(t) - \xi_1(t)$, $x_2(t) - \xi_2(t)$, $x_3(t) - \xi_3(t)$ and $x_4(t) - \xi_4(t)$, respectively. It is observed that the state variable $x(t)$ is effectively estimated by the unknown input observer designed using (Method 1).

2.5.2 Numerical example 2

Consider employing (Method 1) in Section 2.3 to design the output feedback control (2.41) for the controlled plant $G(s)$ written by

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u(t) \\ y(t) & = & \begin{bmatrix} 2 & -11 & -2 & 12 \\ 1 & -16 & -1 & 17 \end{bmatrix} x(t) \end{array} \right. . \tag{2.56}$$

The above controlled plant is of non-minimum phase, since it has an invariant zero at $(20, 0)$.

Using the method in [18], $\tilde{N}(s)$ satisfying (2.9) is obtained as

$$\tilde{N}(s) = \left[\begin{array}{cccc|cc} -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & 0 & 1 \\ \hline 2 & -11 & -2 & 12 & 0 & 0 \\ 1 & -16 & -1 & 17 & 0 & 0 \end{array}\right]. \tag{2.57}$$

The matrix Φ in (2.22), constructed from the state-space representation (2.57), satisfies the condition

$$\text{rank } \Phi = 2, \tag{2.58}$$

as Φ in (2.22) is given by

$$\Phi = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \tag{2.59}$$

with

$$\alpha_1 = 2 \tag{2.60}$$

and

$$\alpha_2 = 1. \tag{2.61}$$

By (2.58), $\hat{\Phi}$ satisfying (2.27) is given by

$$\hat{\Phi} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}. \tag{2.62}$$

Setting

$$T_1 = 0.001, \tag{2.63}$$

$$T_2 = 0.002, \tag{2.64}$$

$$\bar{D}_l = I, \tag{2.65}$$

together with (2.28), (2.29), (2.30), (2.31), (2.33) and (2.34), we have

$$\begin{cases} \beta_{11} = 2000 \\ \beta_{12} = 1000000 \\ \beta_{21} = 500 \end{cases}, \tag{2.66}$$

$$X = \begin{bmatrix} 1000000 & 0 \\ 0 & 500 \end{bmatrix}, \tag{2.67}$$

$$E = I, \tag{2.68}$$

$$\Psi = \begin{bmatrix} 998001 & 0 \\ 0 & 499 \\ 996004 & 0 \\ 0 & 498 \end{bmatrix}, \tag{2.69}$$

$$P = \begin{bmatrix} 0.0869 & -0.0174 & 0.0827 & -0.0166 \\ -0.0174 & 0.0035 & -0.0166 & 0.0033 \\ 0.0827 & -0.0166 & 0.0788 & -0.0158 \\ -0.0166 & 0.0033 & -0.0158 & 0.0032 \end{bmatrix} \tag{2.70}$$

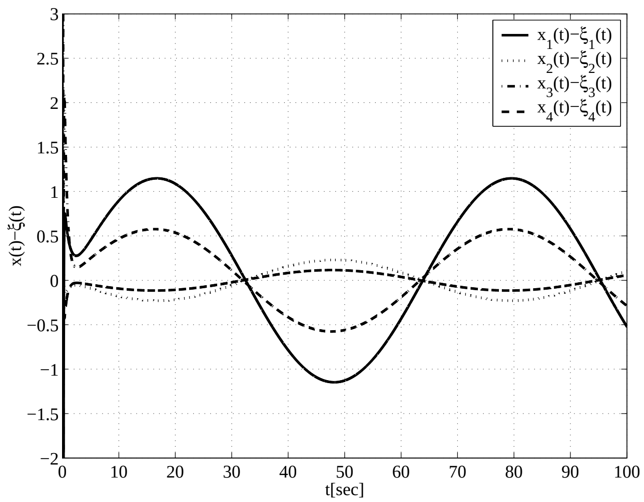

Fig. III: Time response of state estimation error $x(t) - \xi(t)$.

and

$$K = \begin{bmatrix} 0.8298 & -0.3657 \\ -0.3657 & -0.9248 \\ 0.7452 & -0.3484 \\ -0.3484 & -0.9263 \end{bmatrix}. \quad (2.71)$$

Substituting the above parameters into (2.25), $Q(s)$ is obtained. Consequently, the unknown input observer (2.5) reduces to (2.15) in the intended bandwidth of the control system.

When the external input $v(t)$ and initial state $x(0)$ are supplied as the same with (2.43) and (2.44), respectively, the state estimation error $x(t) - \xi(t)$ evolves over time as depicted in Fig. III, where the solid, dotted, alternate long/short dash, broken lines correspond with $x_1(t)-\xi_1(t)$, $x_2(t)-\xi_2(t)$, $x_3(t)-\xi_3(t)$ and $x_4(t)-\xi_4(t)$, respectively.

Fig. III shows that the state variable $x(t)$ is not fully estimated by the unknown input observer designed using (Method 1). The reason why (Method 1) failed is that $G_K(s)$ in (2.16) is not a diagonal inner function. Next, to circumvent this problem, we will design the unknown input observer according to (Method 2).

Let $\tilde{N}(s)$, Φ, $\alpha_i\,(i=1,2)$, $\hat{\Phi}$, $T_i\,(i=1,2)$, $\bar{D}_l$, $\beta_{ij}\,(i=1,2:j=1,\ldots,\alpha_i)$, X, E and Ψ be the same with (2.57), (2.59), (2.60), (2.61), (2.62), (2.63), (2.64), (2.65), (2.66), (2.67), (2.68) and (2.69), respectively. Using these parameters, $G_0(s)$ is determined by (2.36). By obtaining the minimal realization of $G_{0i}(s)\,(i=1,2)$ and calculating $P_i\,(i=1,2)$, we have

$$K_1 = \begin{bmatrix} -1.9010 & 0.3803 & -1.8109 & 0.3623 \end{bmatrix}^{\mathrm{T}} \quad (2.72)$$

and

$$K_2 = \begin{bmatrix} 9.5011 & -1.9010 & 9.0511 & -1.8109 \end{bmatrix}^{\mathrm{T}}. \quad (2.73)$$

Using above parameters, $G_K(s)$ in (2.74) is obtained as

$$G_K(s) = \left[\begin{array}{cc|cc} -20.0000 & 0 & 2.6775 & 0 \\ 0 & -20.0000 & 0 & -2.6255 \\ \hline 14.9393 & 0 & -1 & 0 \\ 0 & -15.2355 & 0 & -1 \end{array}\right] = \begin{bmatrix} \frac{-s+20}{s+20} & 0 \\ 0 & \frac{-s+20}{s+20} \end{bmatrix}. \quad (2.74)$$

It is verified that $G_K(s)$ in (2.74) is a diagonal inner function. In Fig. IV, we depict the state estimation error resulting from the unknown input observer designed using (Method 2), and confirm that the state variable $x(t)$ is effectively estimated.

2.6 Conclusion

In this chapter, we proposed alternative design methods of unknown input observers for the non-minimum phase plant (2.1) by focusing on the intended bandwidth of the control system. The proposed design methods start

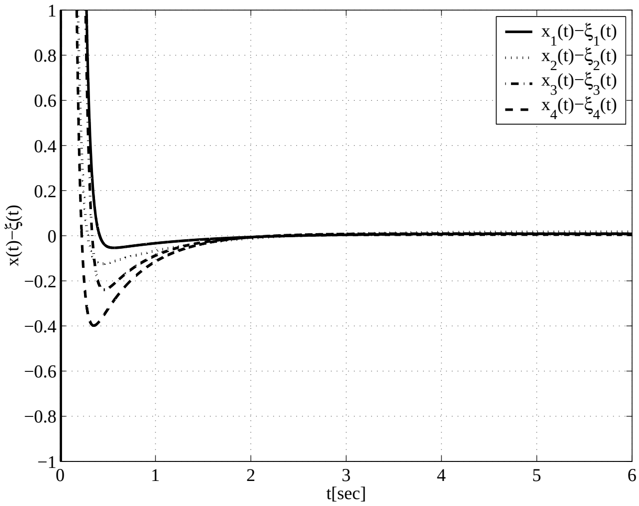

Fig. IV: Time response of state estimation error $x(t) - \xi(t)$.

from the parametrization of all state observers (2.5), (2.7), (2.8), and determine the free-parameter $Q(s)$ by utilizing the techniques of the stable left filtered inverses [20, 21]. The stable left filtered inverses in [20] and [21] led to the two methods (Method 1) and (Method 2), respectively. In Sections 2.4 and 2.5, it is also described that the proposed unknown input observer can be employed for constructing output feedback control systems. Two sample cases were considered in order to illustrate the features of (Method 1) and (Method 2). It was observed that (Method 2) enabled to estimate the state variable effectively even in the case (Method 1) failed. In the recent authors' work [27], the underlying technique of the stable left filtered inverses [20, 21] is extended to a class of nonlinear systems. Hence a future subject of research is to enhance the proposed design methods of unknown input observers to the extent of handling the nonlinear systems directly.

Chapter 3

A design method of disturbance observers for constant disturbances

3.1 Introduction.

In this chapter, we examine the parameterization of all linear disturbance observers using the states and inputs of the system as variables for constant disturbances. Disturbance observers are used in the motion-control field to cancel disturbances or make a closed-loop system robustly stable [28][29][30][31][32][33][34] [35][36]. Generally, the disturbance observer includes the disturbance signal generator and observer. The disturbance, which is usually assumed to be step disturbance, is estimated using the observers. Because the disturbance observer has a simple structure and is easy to understand, it has been used in many applications[28][29][30][31][32][33][34][35][36].

Disturbance observer-based control has been seen as the most promising approach to attenuate disturbances[37]. To linear systems , Li et al. proposed the frequency domain disturbance observer, time domain disturbance observer and extended state observer in different domain. To the nonlinear systems, they proposed nonlinear disturbance observer for constant disturbances and nonlinear disturbance observer for general exogenous disturbances based on different disturbances in[37][38]. But there is no parameterization of all disturbance observers for any disturbance, if the parameterization of all disturbance observers for any disturbance could be obtained, we could express previous studies of disturbance observers in a uniform manner. For solving this problem, Yamada et al. proposed a parameterization of linear disturbance observer for constant disturbances[39]. However, not all parameterizations of linear disturbance observer has been obtained, it is still a disturbance observer for specific disturbances. In addition, the variables of disturbance observer in[39] are the inputs and outputs of system. That is, the parameterizations of linear disturbance observer that uses inputs and states as the variables has not been obtained. Because of the diversity of systems, there are still lots of work to be solved about the parameterization of other various systems.

In this chapter, we propose the parameterization of linear disturbance observers for constant disturbances. By using the states and inputs of the system as variables, the parameterization for constant disturbances was obtained. and the validity of the disturbance observer was proofed by simulation. This chapter is organized as follows: In Section 3.2, the problem considered in this chapter is formulated. then, the structure and necessary characteristics of the linear disturbance observer is defined. In Section 3.3, the parameterization of the linear disturbance observer for constant disturbances is clarified. Then the values of functions are calculated in details. In Section 3.4, a numerical example is presented to show the effectiveness of the proposed parameterization. In Section 3.5, the contributions of this chapter are summerized.

3.2 Problem formulation.

Consider the linear system described by:

$$\begin{cases} \dot{x}(t) &= Ax(t) \quad +Bu(t) \quad +d(t) \\ y(t) &= Cx(t) \end{cases} \tag{3.1}$$

where $x(t) \in R^n$ is the state variable, $u(t) \in R^p$ is the control input, $y(t) \in R^m$ is the output, $d(t) \in R^n$ is the disturbance, $A \in R^{n\times n}$, $B \in R^{n\times p}$, $C \in R^{m\times n}$. It is assumed that $d(t)$ is unavailable but constant one, that is

$$d(t) = d(= const.) \tag{3.2}$$

and $u(t)$ and $x(t)$ are available.

When the disturbance $d(t)$ is not available, a disturbance estimator called the disturbance observer is frequently used. The disturbance observer estimates the disturbance $d(t)$ in (3.1) using available signal. From the assumption that the available signal are $u(t)$ and $x(t)$ in this chapter. Thus, the general form of the disturbance observer $\tilde{d}(s)$ for (3.1) is written as:

$$\tilde{d}(s) = F_1(s)X(s) + F_2(s)U(s) \tag{3.3}$$

where $F_1(s) \in R^{n\times n}(s)$, $F_2(s) \in R^{n\times p}(s)$, $\tilde{d}(s) = \mathcal{L}(\tilde{d}(t))$, $\tilde{d}(t) \in R^n(t)$, $X(s) = \mathcal{L}(x(t))$ and $U(s) = \mathcal{L}(u(t))$. We call the system $\tilde{d}(s)$ in (3.3) a disturbance observer for constant disturbances, if:

$$\lim_{t\to\infty}\left(d(t) - \tilde{d}(t)\right) = 0 \tag{3.4}$$

is satisfied for any $x(0)$, $u(t)$ and $d(t)$.

The problem considered in this chapter is to obtain the the parameterization of all linear disturbance observers $\tilde{d}(s)$ in (3.3) for constant disturbances in (3.2).

3.3 Parameterization of all linear disturbance observers for constant disturbances

The parameterization of all linear disturbance observers $\tilde{d}(s)$ in (3.3) for constant disturbances is summarized in the following theorem.

Theorem 3.3.1 *The system $\tilde{d}(s)$ in (3.3) is a disturbance observer for constant disturbances if and only if $F_1(s)$ and $F_2(s)$ are written by:*

$$F_1(s) = (I - Q(s))\,(sI - A) \tag{3.5}$$

and

$$F_2(s) = -\,(I - Q(s))\,B, \tag{3.6}$$

respectively, where $Q(s) \in RH_\infty$ is any function that makes $(I - Q(s))(sI - A)$ proper and $Q(0) = 0$.

(Proof)
First, the necessity is shown. That is, we show that if the system $\tilde{d}(s)$ in (3.3) satisfies (3.4), then $F_1(s)$ and $F_2(s)$ in (3.3) are written by (3.5) and (3.6), respectively.

From the assumption in (3.4), $d(s) - \tilde{d}(s)$ is written by the form as

$$d(s) - \tilde{d}(s) = Q(s)d(s), \tag{3.7}$$

where $Q(s) \in RH_\infty$.

From the final value theorem of Laplasse transformation, (3.7) and (3.2), we have

$$\begin{aligned}\lim_{t\to\infty}(d(t) - \tilde{d}(t)) &= \lim_{s\to 0} s\left(d(s) - \tilde{d}(s)\right)\\ &= \lim_{s\to 0} sQ(s)d(s)\\ &= \lim_{s\to 0} sQ(s)\frac{d}{s}\\ &= Q(0)d.\end{aligned} \tag{3.8}$$

From this equation and (3.4), $Q(s)$ need to satisfy

$$Q(0) = 0. \tag{3.9}$$

Equation (3.7) can be rewritten as:

$$\tilde{d}(s) = (I - Q(s))\,d(s). \tag{3.10}$$

From (3.1), $d(s)$ is written by

$$d(s) = (sI - A)X(s) - BU(s). \tag{3.11}$$

Substituting (3.11) for (3.10), we have:

$$\begin{aligned}&\tilde{d}(s)\\ &= (I - Q(s))\,((sI - A)X(s) - BU(s))\\ &= (I - Q(s))\,(sI - A)X(s) - (I - Q(s))\,BU(s).\end{aligned} \tag{3.12}$$

In order to satisfy (3.4), $F_1(s)$ and $F_2(s)$ need to be proper. Therefore it is necessary that $(I - Q(s))\,(sI - A)$ and $(I - Q(s))B$ are proper. Since $(I - Q(s))B$ is obviously proper, $Q(s) \in RH_\infty$ is any function that makes $(I - Q(s))\,(sI - A)$ proper and $Q(0) = 0$. We have thus proved the necessity.

Next, the sufficiency is shown. That is, we show that if $F_1(s)$ and $F_2(s)$ are described by (3.5) and (3.6), and $Q(s) \in RH_\infty$ is any function that makes $(I - Q(s))(sI - A)$ proper and $Q(0) = 0$, $\tilde{d}(s)$ in (3.3) satisfies (3.4).

Substituting (3.5) and (3.6) for (3.3), $\tilde{d}(s)$ is written by:

$$\tilde{d}(s) = (I - Q(s))(sI - A)X(s) - (I - Q(s))BU(s). \tag{3.13}$$

$d(s) - \tilde{d}(s)$ satisfies:

$$\begin{aligned} & d(s) - \tilde{d}(s) \\ = \ & d(s) - (I - Q(s))(sI - A)X(s) + (I - Q(s))BU(s) \\ = \ & d(s) - (I - Q(s))((sI - A)X(s) - BU(s)) \\ = \ & d(s) - (I - Q(s))d(s) \\ = \ & Q(s)d(s). \end{aligned} \tag{3.14}$$

From the final value theorem of Laplasse transformation and $Q(0) = 0$, we have:

$$\begin{aligned} \lim_{t\to\infty}(d - \tilde{d}(t)) &= \lim_{s\to 0} s\left(d(s) - \tilde{d}(s)\right) \\ &= \lim_{s\to 0} sQ(s)d(s) \\ &= \lim_{s\to 0} sQ(s)\frac{d}{s} \\ &= Q(0)d \\ &= 0 \end{aligned} \tag{3.15}$$

In this way,sufficiency has been proved.

We have thus proved this theorem.

3.4 Numerical example.

In this section, a numerical example is shown to illustrate the effectiveness of the proposed method.

Consider the problem of obtaining the parameterization of all linear disturbance observers for constant disturbances for the unstable plant written by:

$$\left\{ \begin{array}{rcl} \dot{x}(t) &=& \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) + d(t) \\ y(t) &=& \begin{bmatrix} 1 & -3 \end{bmatrix} x(t) \end{array} \right. \tag{3.16}$$

For the plant in (3.16), from Theorem 3.3.1, the parameterization of all disturbance observer is written by (3.3), where $F_1(s)$ and $F_2(s)$ are written by (3.5) and (3.6), where $Q(s) \in RH_\infty$ is any function that makes $(I - Q(s))(sI - A)$ proper and $Q(0) = 0$.

We settle $Q(s)$ in (3.5) and (3.6) as

$$Q(s) = \frac{1}{(s+3)s+2} \begin{bmatrix} s^2 & -2s \\ s & s^2 + 3s \end{bmatrix}. \tag{3.17}$$

$Q(s)$ in (3.17) is obviously belong to RH_∞ and satisfy $Q(0) = 0$. In addition, $(I - Q(s))(sI - A)$ can be expressed as:

$$\begin{aligned} & (I - Q(s))(sI - A) \\ = \ & \frac{1}{s^2 + 3s + 2} \begin{bmatrix} 3s^2 + 3s + 2 & 2s^2 - 6s - 4 \\ -s^2 - s - 2 & 4s \end{bmatrix} \end{aligned} \tag{3.18}$$

This equation means that $Q(s)$ in (3.17) makes $(I - Q(s))(sI - A)$ proper.

Using $Q(s)$ in (3.17), $F_1(s)$ in (3.5) and $F_2(s)$ in (3.6) are given by

$$F_1(s) = \frac{1}{(s+3)s+2} \begin{bmatrix} 3s^2 + 3s + 2 & 2s^2 - 6s - 4 \\ -s^2 - s - 2 & 4s \end{bmatrix} \tag{3.19}$$

and

$$F_2(s) = \frac{-1}{(s+3)s+2} \begin{bmatrix} 3s + 2 \\ -s \end{bmatrix}, \tag{3.20}$$

respectively

When the control input $u(t)$ is given by:

$$u(t) = 1$$

and the disturbance $d(t)$ are given as:

$$d(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \tag{3.21}$$

$d(t)$ are shown in Fig. I. Here the solid line shows the disturbance of $d_1(t)$ and the dotted line shows that of $d_2(t)$.

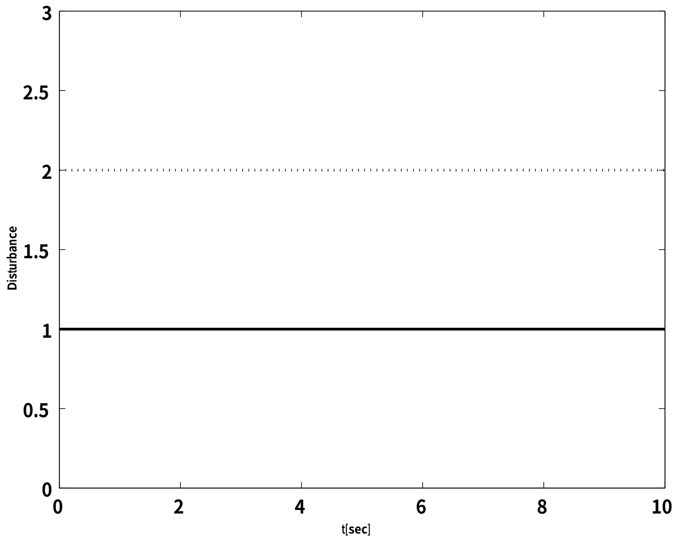

Fig. I: The constant disturbances

and the response of

$$\begin{aligned} \tilde{d}(t) &= \mathcal{L}^{-1}\{\tilde{d}(s)\} \\ &= \begin{bmatrix} \tilde{d}_1(t) \\ \tilde{d}_2(t) \end{bmatrix} \end{aligned} \tag{3.22}$$

is shown in Fig. II. Here the solid line shows the response of $\tilde{d}_1(t)$ and the dotted line shows that of $\tilde{d}_2(t)$. Figure II shows that the disturbance observer in (3.3) can estimate the constant disturbance.

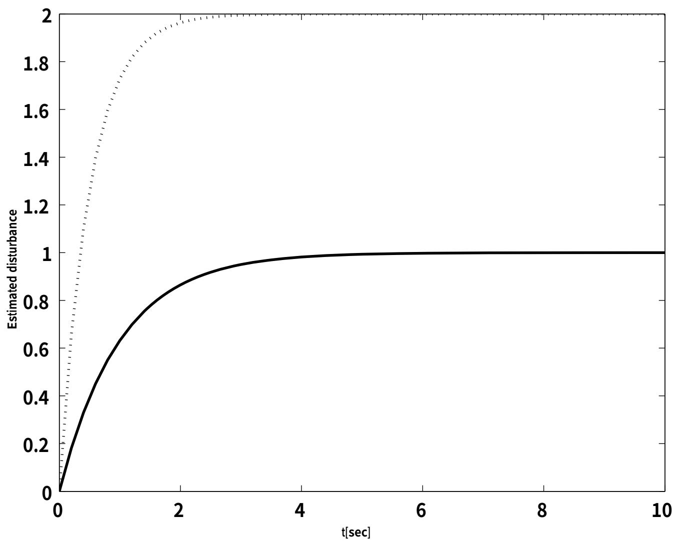

Fig. II: Response of the estimated disturbances

The response of the error $e(t)$ written by

$$e(t) = \begin{bmatrix} d_1(t) - \tilde{d}_1(t) \\ d_2(t) - \tilde{d}_2(t) \end{bmatrix} = \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} \tag{3.23}$$

is shown in Fig. III. Here the solid line shows the response of $e_1(t)$ and the dotted line shows that of $e_2(t)$. Figure III shows that the disturbance observer can estimate the disturbance $d(t)$ effectively.

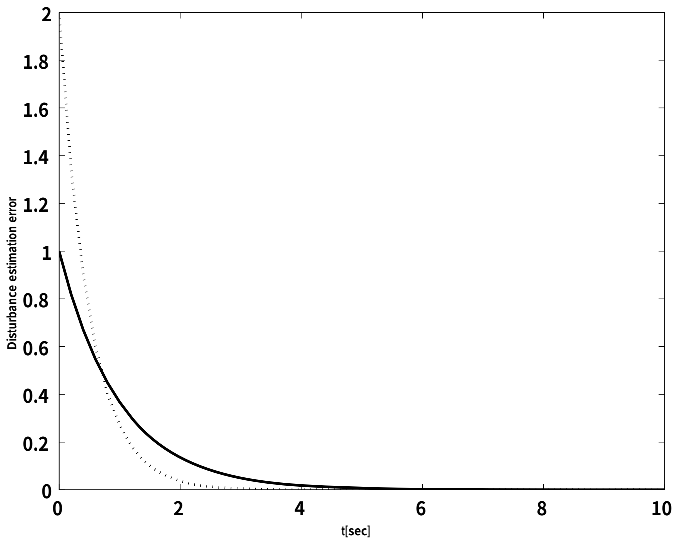

Fig. III: Response of the errors $e(t) = d(t) - \tilde{d}(t)$

In this way, we find that using the parameterization of all disturbance observer for constant disturbances in Theoremthe:ob, we can design a linear disturbance observer easily.

3.5 Conclusions.

In this chapter, We have proposed the parameterization of all linear disturbance observers for constant disturbances. A numerical example confirmed the validity of the analysis. From the numerical example we can see that although it takes five seconds to estimate the constant disturbances thoroughly, the disturbance observer can still estimate the constant disturbances accurately in final.

The problem to design disturbance observers for time-varying disturbance will be considered in another article.

Chapter 4

Conclusion

In this thesis, we propose a design method of unknown input observer for non-minimum phase plants and a parameterization of all linear disturbance observers using the states and input of the system as variables for constant disturbances were proposed.

In chapter 2, we proposed a design method of unknown input observer for non-minimum phase plants. Furthermore, as a complement to the proposed design methods, we described that the resulting unknown input observers can be employed for constructing output feedback control systems if it is combined with the H_∞ state feedback control [22]. In addition, A numerical example was presented to show that a design method of unknown input observer for non-minimum phase plants.

In chapter 3, we proposed the parameterization of linear disturbance observers for constant disturbances. By using the states and inputs of the system as variables, the parameterization for constant disturbances was obtained. A numerical example was presented to show the effectiveness of the proposed parameterization.

In future work, the proposed design methods of unknown input observers to the extent of handling the non-linear systems and the problem to design disturbance observers for time-varying disturbance will be considered.

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Bibliography

[1] D. G. Luenberger, Observing the state of a linear system, *IEEE Trans. Mil. Electron*, vol. 8, pp. 74–80, 1964.

[2] D. G. Luenberger, Observers for multivariable systems, *IEEE Trans. on Automatic Control*, vol. 11, pp. 190–197, 1966.

[3] D. G. Luenberger, An introduction to observers, *IEEE Trans. on Automatic Control*, vol. 16, pp. 596–602, 1971.

[4] G. C. Goodwin and R. H. Middleton, The class of all stable unbiased state estimators, *Systems & Control Letters*, vol. 13, pp. 161–163, 1989.

[5] X. Ding, L. Guo and P. M. Frank, Parametrization of linear observers and its application to observer design, *IEEE Trans. on Automatic Control*, vol. 39, pp. 1648–1652, 1994.

[6] Y. W. Kim, G. Rizzoni and Y.-Y. Wang, Design of an IC engine torque estimator using unknown input observer, *Transactions of the ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 121, pp. 487–495, 1999.

[7] Z. Kang, S. Fujii, C. Zhou and K. Ogata, Adaptive control of a planar gantry crane by the switching of contrrollers, *Transactions of the Society of Instrument and Control Engineers*, vol. 35, pp. 253–261, 1999.

[8] P. Kudva, N. Viswanadham and A. Ramakrishna, Observers for linear systems with unknown inputs, *IEEE Trans. on Automatic Control*, vol. 25, pp. 113–115, 1980.

[9] K. K. Busawon and P. Kabore, Disturbanc attenuation using proportional integral observers, *International Journal of Control*, vol. 74, pp. 618–627, 2001.

[10] A. Saberi, A.A. Stoorvogel and P. Sannuti, Exact, almost and optimal input decoupled (delayed) observers, *International Journal of Control*, vol. 73, pp. 552–581, 2001.

[11] P. L. Hsu, Y. C. Houng and S. S. Yeh, Design of an optimal unknown input observer for load compensation in motion systems, *Asian Journal of Control*, vol. 3, pp. 204–215, 2001.

[12] J. Jin, M. J. Tahk and C. Park, Time-delayed state and unknown input observation, *International Journal of Control*, vol. 66, pp. 733–745, 1997.

[13] F. Amato and M. Mattei, Design of full order unknown input observers with H_∞ performance, *Proceedings of the 2002 IEEE International Conference on Control Applications*, Scotland, pp. 74–75, 2002.

[14] R. Suzuki, M. Tani, D. Yamashita and N. Kobayashi, Disturbance decoupling control of a mechanical system by reduced order observer based stabilizing controller, *Proceedings of the 2004 IEEE International Conference on Control Applications*, Taiwan, pp. 1751–1756, 2004.

[15] T. Mita, On the synthesis of an unknown input observer for a class of multi-input/output systems, *International Journal of Control*, vol. 26, pp. 841–851, 1977.

[16] H. Hikita, A solution of an exact model matching problem and an unknown input observer by transfer function approach, *Transactions of the Society of Instrument and Control Engineers*, vol. 16, pp. 635–642, 1980.

[17] K. Fuwa, T. Narikiyo, M. Ishida and H. Kandoh, Observer synthesis for linear time invariant systems with unknown inputs via eigenstructure assignment and its application to disturbance attenuation, *Transactions of the Society of Instrument and Control Engineers*, vol. 43, pp. 232–240, 2006.

[18] C. N. Nett, C. A. Jacobson and M. J. Balas, A connection between state-space and doubly coprime fractional representation, *IEEE Trans. on Automatic Control*, vol. 29, pp. 831–832, 1984.

[19] M. Vidyasagar, *Control system synthesis: A factorization approach*, MIT Press, London, 1985.

[20] K. Yamada, K. Watanabe and Z. B. Shu, A state space design method of stable filtered inverse systems and its application to H_2 suboptimal internal model control, *Proceedings of International Federation of Automatic Control World Congress'96*, San Francisco, pp. 379–382, 1996.

[21] K. Yamada and W. Kinoshita, New design method of stable filtered inverse systems, *Proceedings of 2002 American Control Conference*, Anchorage, pp. 4738–4743, 2002.

[22] T. Mita, K.Z. Liu and S. Ohuchi, Correction of the FI results in H_∞ control and parametrization of H_∞ state feedback controllers, *IEEE Trans. on Automatic Control*, vol. 38, pp. 343–347, 1993.

[23] K. Yamada and M. Kobayashi, A design method for unknown input observer for non-minimum phase systems, *Proceedings of 4th International Conference on Mechatronics and Information Technology-Mechatronics, MEMS, and Smart Materials*, vol. 6794, 2007.

[24] A. Termehchy and A. Afshar, A novel design of unknown input observer for fault diagnosis in non-minimum phase systems, *Proceedings of the 19th World Congress*, pp. 8552–8557, 2014.

[25] A. I. Malikov, State and unknown inputs finite time estimation for time-varying nonlinear Lipschitz systems with uncertain disturbances, *Proceedings of 20th IFAC World Congress*, pp. 1439–1444, 2017.

[26] B. Marx, D. Ichalal, J. Ragot, D. Maquin and S. Mammar, Unknown input observer for LPV systems, *Automatica*, vol. 100, pp. 67–74, 2019.

[27] Y. Kimura, K. Hashikura, T. Suzuki and K. Yamada, State space design method for left filtered inverse systems for non-linear systems, *ICIC Express Letters*, vol. 13, pp. 493–497, 2019.

[28] K. Ohishi, K. Ohnishi and K. Miyachi, *Torque-speed regulation of DC motor based on load torque estimation.* Proc.IEEJ IPEC-TOKYO, Vol.2, pp.1209-1216, 1983.

[29] K. Ohishi, K. Ohnishi, S. Komada and K. Ohnishi, *Force feedback control of robot manipulator by the acceleration tracing orientation method.* IEEE Transactions on Industrial Electronics, Vol.37, No.1, pp.6-12, 1990.

[30] T. Umeno and Y. Hori, *Robust speed control of DC servomotors using modern two degrees-of-freedom controller design.* IEEE Transactions on Industrial Electronics, Vol.38, No.5, pp.363-368, 1991.

[31] M. Tomizuka and, *On the design of digital tracking controllers.* Transactions of the ASME Journal of Dynamic Systems, Measurement,and Control, Vol.115, pp.412-418, 1993.

[32] K. Ohnishi, M. Shibata and T. Murakami, *Motion control for advanced mechatronics.* IEEE/ASME Transaction on Mechatronics, Vol.1, No.1, pp.56-57, 1996.

[33] H. S.Lee and M. Tomizuka, *Robust motion controller design for high-accuracy positioning systems.* IEEE Transactions on Industrial Electronics, Vol.43, No.1, pp.48-55, 1996.

[34] T. Mita, M. Hirata, K. Murata and H. Zhang, *Control versus disturbance-observer-based control.* IEEE Transactions on Industrial Electronics, Vol.45, No.3, pp.488-495, 1998.

[35] H. Kobayashi S. Katsura and K. Ohnishi, *An analysis of parameter variations of disturbance observer for motion control.* IEEE Transactions on Industrial Electronics, Vol.54, No.6, pp.3413-3421, 2007.

[36] K. Ohishi, *Realization of fine motion control based on disturbance observer.* Proc.of AMC'08, 2008.

[37] S. H.Li, J. Yang, W. H.Chen and X. S.Chen, *Disturbance Observer-Based Control.* CRC Press, pp.43-52, 2014.

[38] W. H.Chen, D. J.Ballance, P. J.Gawthrop and P. O'Reilly, *A nonlinear disturbance observer for robotic manipulators..* IEEE Transactions on Industrial Electronics, Vol.47, No.4, pp.932-935, 2000.

[39] K. Yamada, I. Murakami, Y. Ando, T. Hagiwara, Y. Imai and M. Kobayashi, *The parameterization of all disturbance observers.* ICIC Express Letter, Vol.2, No.4, pp.421-426, 2007.

Publication papers

Chapter 2	○	J. Juntawongso, M. Kobayashi, K. Hashikura, M. A. S. Kamal and K. Yamada, "STATE SPACE DESIGN METHOD FOR UNKNOWN INPUT OBSERVERS", International Journal of Innovative Computing, Information and Control, Vol. 17, No. 1, pp. 153-165, 2021.
Chapter 3	○	J. Juntawongso, C. F. Zang, K. Hashikura, T. Suzuki and K. Yamada, "Disturbance observers for constant disturbances," 2019 16th International Conference on Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology (ECTI-CON), Pattaya, Chonburi, Thailand, pp. 850-853, 2019